MANGALORE UNIVERSITY
DEPARTMENT OF POST-GRADUATE STUDIES AND RESEARCH IN MATHEMATICS

Syllabus for Ph. D. Course Work in Mathematics

a) Course Pattern

The course work shall be of the following pattern:

Papers	Particulars	Hours of Instruction per week	Duration of Examination (hrs)	Marks			Credits
				IA	Theory	Total	
Paper 1	Research Methodology	4	3	30	70	100	4
Paper 2	Theoretical Foundations	4	3	30	70	100	4
Paper 3	Recent Developments	4	3	30	70	100	4
Paper 4	Reviewing of Literature and Planning of the Proposed Research Work with a Tentative Title	16	-	-	-	200	8
		Total					20 Credits

b) **Part-time researchers may be allowed to complete the course work in two semesters. They shall take the Papers 1 to 3 in the first semester and Paper 4 in the second semester.**

c) **The candidates are required to undertake the course work for a semester (six months duration) immediately after enrollment as per the Calendar notified by the Registrar.**

d) **Internal assessment (IA) marks for each of the papers Paper 1 to 3 shall be based on at least one written test and one seminar.**
Pattern of Semester Examination Question Papers for each of the papers Paper 1 to 3: **Each question paper shall contain Part A and Part B. Part A shall contain 10 short answer questions of 2 marks each out of which 7 questions are to be answered. Part B shall contain 8 questions of 14 marks each out of which 4 questions are to be answered.**

Paper 1 : Research Methodology (common to all candidates):

1. **Review and update of basic concepts in Algebra:**

   Examples of Groups and General Theory, S_n, A_n, D_n. Direct products, Free Groups, Products, Generators and Relations, Finite Groups. Rings and Ideals, Polynomials, Fields and their Extensions, Elementary Number theory.

   Vector Spaces, Rank and Determinants, Systems of Equations, Linear Transformations, Eigen values and Eigen vectors, Canonical forms, Similarity, Bilinear, Qudratic forms and Inner product spaces, General theory of matrices.

2. **Review and update of basic concepts in Real and Complex Analysis:**

   Limits and Continuity, Sequences, Series and Products, Differential Calculus, Integral Calculus, Sequences of functions, Fourier Series, Convex Functions.

   Conformal mappings, Integral representation of analytic functions, Functions on the unit disc, Analytic and Meromorphic functions. Zeros and singularities, Harmonic functions and Residue theory.

3. **Review and update of basic concepts in Topology:**

   Topology of R^n, General theory, Fixed point theorem.

References:

1. Paulo Ney de Souza, Jorge-Nuno Silva, *Berkeley Problems in Mathematics,* Springer-Verlag,1998.
2. Michael Artin, *Algebra*, Prentice-Hall of India.
3. Walter Rudin, *Principles of Mathematical Analysis*, McGraw Hill, 3rd edition.
4. Walter Rudin, *Real and Complex Analysis,* McGraw Hill International Edition, New Delhi, 1987.
5. J.R.Munkres,*Topology*, Prentice-Hall of India, 1975, 2nd edition, 2000.

Paper 2 : Theoretical Foundations (common to all candidates):

1. **Algebra** - Group Representations. Definition of a Group Representation – G-Invariant forms and Unitary Representations – Compact groups – G-invariant subspaces and Irreducible Representations – Characters – Permutation Representation and Regular

Representation – One Dimensional Representations – Shur's Lemma and proof of the orthogonality relations – Representations of the group SU_2.

2. **Analysis** – Fourier Transforms – Formal properties – The inversion theorem – The plancheral Theorem – The Branch algebra L^1 – The Holomorphic Fourier transform – Two theorems of Paley and Wiener – Quasi analytic class – The Denjoy – Carlemen Theorem.
   Manifolds : Manifolds – Differentiable manifolds – Differential forms – Integration.
3. **Topology** : Fundamental group and covering spaces – Simplicial complexes – Geometry of simplicial complexes – Bary centre subdivision – Simplicial approximation theory.

References:

1. Michael Artin, *Algebra*, Prentice Hall of India.
2. J. P. Serre ,*Linear Representations of finite groups* – Springer – Verlag- New York
3. Walter Rudin, *Real and Complex Analysis*, McGraw Hill International Edition, New Delhi, 1987.
4. N. Wiener, The *Fourier Integral and Certain of its Applications*, Dover Publ. Inc. New York.
5. L.Auslander and R.F. Mackenzie, *Introduction to Differentiable Manifolds*, McGraw-Hill, New York, 1963.
6. I. M. Singer and J.A. Thorpe, *Lecture Notes on Elementary Topology*, Springer Verlag, New York, 1967.
7. J.G. Hocking and G.S. Young, *Topology*, Addison – Wesley. Pub Co., Mass 1961.

Paper 3 : Recent Developments:

Any **one** of the following two elective subjects shall be chosen by the candidate as per the choice of specialization:

Elective - 1 : Lattice Theory

Elective - 2 : Graph Theory

Elective - 1 : Lattice Theory

1. Partially ordered sets:

Axiom of choice, Zorn's lemma, Hausdorff's maximal chain principle and well ordering theorem. Ordinal sums of posets, Direct products of posets.

2. Lattices in general :

Congruence relations and congruence lattices of lattices. Direct products and congruence relations. Polynomials, Identities and Inequalities in lattices.

3. Complete lattices:

Characterization of completeness in terms of the fixed point property. Compactly generated lattices, weak atomicity and upper continuity. Galois connections, Dedekind cuts.

4. Distributive and modular lattices:

Meet representations in modular and distributive lattices. Independent sets and direct joins, Ore's theorem on the direct joins for modular lattices of finite length. Pseudo complemented lattices, Stone algebras. Congruence relations and description of principal congruence relations in distributive lattices. Hashimoto's theorem regarding ideals and congruence relations of lattices. Infinitely distributive and completely distributive lattices. Distributive sublattices of modular lattices.

5. Boolean algebras:

Boolean algebras and lattice of propositions. Complete Boolean algebras and complete distributivity.

6. Semimodular lattices:

M-symmetric lattices and Semimodular lattices. Geometric lattices. Partition lattices of sets.

References:

1. G.Szasz, *Introduction to lattice theory*, Academic Press, NewYork 1963.
2. L.A.Skornjakov, *Elements of lattice theory*, Hindustan Publishing Corporation, 1977.
3. G.Gratzer, *General lattice theory*, Birkhauser Verlag, Basel, 2000.

4. P.Crawley and R.P.Dilworth, *Algebraic theory of lattices*, Prentice- Hall Inc. N.J.1973.
5. G.Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, Volume 25, 1995.
6. F.Maeda and S.Maeda, *Theory of symmetric lattices*, Springer Verlag, New York, 1970.
7. M.Stern, *Semimodular lattices*, Cambridge University Press, 1999.
8. B.A.Davey and H.A.Priestley, *Introduction to lattices and order*, Cambridge University Press, Cambridge 1990.
9. R.Balbes and Ph. Dwinger, *Distributive lattices*, University of Missouri Press, 1974.

Elective - 2 : Graph Theory

1. Graphs and Subgraphs:

Graphs, Subgraphs, Special Graphs, Operations on graphs, Degree sequence and degree sets

2. Connected and disconnected graphs:

Paths and cycles, complementary graphs, Cut vertices and bridges, Eulerian graphs, Blocks, Critical and minimal graphs, Centers, Connectivity, Hamiltonicity

3. Planar and Nonplanar graphs:

Euler's formula, Nonplanar graphs, Homeomorphism and contraction, Characterization of planar graphs, Outerplanar graphs

4. Matching, Factorization and coverings:

Matchings, Factorization, Coverings

5. Graph Isomorphism and Reconstruction:

Line graphs, Reconstruction problem, spectrum of a graph

6. Networks:

Flows and cuts, The Max-Flow Min-cut Theorem, Applications of Max-Flow Min-cut Theorem

7. Graph Embedding:

The genus of a graph, 2-cell embedding of graphs, the maximum genus of a graph.

8. Domination in graphs:

The Domination number of a graph, the independent domination number of a graph, other domination parameters.

9. Extremal graph theory:

Turan's theorem, extremal results in graphs, cages.

10. Matrix representation of a graph G:

Incidence matrix $B(G)$, Sub matrices of adjacency matrix $A(G)$, Circuit matrix $C(G)$, Fundamental circuit matrix & rank of $B(G)$. An application to switching Network. Cut-set matrix, Relation between A, B & C, Path matrix, Adjacency matrix

11. Design Methodology of Topological Structure of interconnection networks

Line graphical methods, Cartesian Product Methods, Cayley methods.

References:

1. *Graphs and Digraphs – M. Behzad, G. Chartrand and L. Lesniak-Foster, Publisher: Wadsworth International Group, Belmont, 1979.*
2. *Graphs & Digraph: G. Chartrand & L. Lesniak, 4th edition Chapman & Hall/ CRC.*
3. *Topological Structure and Analysis of Interconnection Networks, Junming Xu, Kluwer Academic publishers*
4. *Fundamentals of Domination in Graphs, Teresa H. Haynes, S. T. Hedetniemi and P. J. Slate.*
5. *Distance in Graphs, F. Buckley and F. Harary, Adision Wesley Publishing Company.*

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Paper 3: Recent Developments

The following is the third elective subject available for choice by candidates in addition to Elective 1: Lattice Theory and Elective 2: Graph Theory, from the year 2014.

Elective-3: Differential Geometry

Elementary differential geometry

Curves in E^3, Covariant derivatives, Frame fields, Connection forms, Structural equations.

Riemannian geometry:

Differential Manifolds, Tangent bundle, Immersions and Embeddings, Examples.

Riemannian metrics, Existence theorem, Affine Connections, Riemannian Connections. Riemannian Curvature, Ricci Curvature, Scalar Curvature, Tensors on Riemannian Manifolds, Einstein Manifolds.

Submanifolds:

Submanifolds of Riemannian Manifolds. Gauss Curvature equation, Codazzi-Mainardi equation. Hypersurfaces of Riemannian Manifolds.

References

1. Elementary Differential Geometry, Barrett O'Neill, Second Edition, Academic Press, 2006.
2. N J Hicks, Notes on Differential Geometry, Van-Nostrand, 1969.
3. M P Do Carmo, Riemannian Geometry, Birkhauser, 1992.
4. D E Blair, Contact Manifolds in Riemannian Geometry, Lecture notes in Mathematics, 509, Springer-Verlag, Berlin, 1976.
5. K.S. Amur, D.J. Shetty, C.S. Bagewadi, An Introduction to Differential Geometry, Narosa, 2010.

Approved by P.G.B.O.S. in mathematics

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