MANGALORE UNIVERSITY

DEPARTMENT OF MATHEMATICS

MSC MATHEMATICS

MTS 405	Number Theory	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts and development of Elementary Number Theory using axioms, definitions, examples, theorems and their proofs.

Course Specific Outcome: At the end of the course students will have the knowledge and skills to:

- Apply the method of solving Linear Diophantine equations, Primality testing and factorization.
- Find the Dirichlet product of arithmetical functions, Dirichlet inverses.
- Solve the Linear congruences, Polynomial congruences modulo p, Simultaneous linear congruences, Simultaneous non-linear congruences, and Solving congruences modulo prime powers.
- Apply the properties of Legendre's symbol , Gauss lemma,
- Understanding the Pythagorean triples and their classification, Fermat's Last Theorem.
- Solving Pell's equation by continued fractions.

Unit I

Divisibility and Primes: Recapitulation of Division algorithm, Euclid's algorithm, Least Common Multiples, Linear Diophantine equations. Prime numbers and Prime-power factorisations, Distribution of primes, Fermat and Mersenne primes, Primality testing and factorization.

Arithmetical Functions: The Mobius function and its properties, Euler function, examples and properties, The Dirichlet product of arithmetical functions, Dirichlet inverses and the Mobius inversion formula.

(12 Hours)

Unit II - Congruences:

Recapitulation of basic properties of congruences, Residue classes and complete residue systems, Linear congruences. Reduced residue systems and the Euler-Fermat theorem, Polynomial congruences modulo p and Langrange's theorem, Simultaneous linear congruences, Simultaneous non-linear congruences, An extension of Chinese Remainder Theorem, Solving congruences modulo prime powers.

(12 Hours)

Unit III - Quadratic Residues and Quadratic Reciprocity Law:

Quadratic residues, Legendre's symbol and its properties, Euler's criterion, Gauss lemma, The quadratic reciprocity law and its applications, The Jacobi symbol, Applications to Diophantine equations.

(12 Hours)

Unit IV - Sums of squares, Fermat's last theorem and Continued fractions:

Sums of two squares, Sums of four squares, The Pythagoras theorem, Pythagorean triples and their classification, Fermat's Last Theorem (Case $n = 4$).

Recapitulation of Finite continued fractions, Infinite continued fractions, Representation of irrational numbers, Periodic continued fractions and quadratic irrationals. Solution of Pell's equation by continued fractions.

(12 Hours)

References

[1] G. A. Jones and J. M. Jones, *Elementary Number Theory*, Springer UTM, 2007.

[2] Tom M. Apostol, *Introduction to Analytic Number Theory*, Springer, 1989.

[3] David M. Burton, *Elementary Number Theory*, 7th Ed., McGraw-Hill, 2010.

[4] Niven, H.S. Zuckerman & H.L. Montgomery, *Introduction to the Theory of Numbers*, Wiley, 2000.

[5] H. Davenport, *The Higher Arithmetic*, Cambridge University Press, 2008.