MANGALORE UNIVERSITY

DEPARTMENT OF MATHEMATICS

MSC MATHEMATICS

MTS 558	Calculus of Variations and Integral Equations	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Course Outcome: Students will have the knowledge and skills toapplythe concepts of the course in – solving difficult popular problems arising in Physics, Chemistry, Engineering and technology, Statistical Analysis, and also in Economics.

Course Specific Outcome:

At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Solving the problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals,
- Solving Variational problems with the fixed boundaries, andMoving boundary problems
- One-sided variations, conditions for one sided variations.
- Variational problems involving conditional extremum, constraints involving several variables and their derivatives, Isoperimetric problems.
- the Conversion of Volterra Equation to ODE, IVP and BVP to Integral Equation.
- the Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigen function expansion, Hilbert-Schmidt theorem.

Unit I - Variational Problems with the Fixed Boundaries:

Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional. Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the

system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples.

(12 Hours)

Unit II - Variational Problems with Moving Boundaries, Sufficiency Conditions:

Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction of extremals, examples, One-sided variations, conditions for one sided variations. Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving severalvariables and their derivatives, Isoperimetric problems, examples.

(12 Hours)

Unit III - Integral Equations:

Introduction, Definitions and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation.Fredholm's Integral equations - Decomposition, direct computation, Successive approximation,Successive substitution methods for Fredholm Integral Equations.

(10 Hours)

Unit IV

Voltera Integral Equations: A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel.

Fredholm's theory - Hilbert-Schmidt Theorem: Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigenfunction expansion, Hilbert-Schmidt theorem.

Fredholm and Volterra Integro-Differential Equation: Fredholm and Volterra Integro-Differential equation, Singular and nonlinear Integral Equation.

(14 Hours)

References

[1] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol I, Interscience Press, 1953.

[2] L. E. Elsgolc, *Calculus of Variations*, Pergamon Press Ltd., 1962.

[3] R. Weinstock, *Calculus of Variations with Applications to Physics and Engineering*, Dover, 1974.

[4] D. Porter and D. S. G. Stirling, Integral Equations, *A practical treatment from spectral theory and applications*, Cambridge University Press, 1990.

[5] R. P. Kanwal, *Linear Integral Equations Theory and Practise*, Academic Press 1971.

[6] A. M. Wazwaz, *Afirst course in integral equations*, World Scientific Press, 1997.

[7] C. Cordumeanu, *Integral Equations and Applications*, Cambridge University Press, 1991.