MANGALORE UNIVERSITY

M.Sc. Mathematics Choice Based Credit System (Semester Scheme) Programme from the academic year 2019-20

(Semester Scheme)

Preamble:

The syllabi for the M.Sc. Mathematics Choice Based Credit System (Semester Scheme) Programme in use at present were introduced from the academic year 2016-17. To enable the programmes to be on par with global standards and to provide hands on experience, Lab components have been added. Hence the following revised and restructured syllabi for the M.Sc. Mathematics Programme have been prepared as per the regulations of the University. The Practical Lab introduced are of 2 credits each in first 3 semesters. In the syllabi, all the hard core and soft core courses have been retained from the syllabus of 2016-17. The first paper in each of the second and the third semesters is an "Open Elective" paper, which is offered only to the students of other departments. The syllabi takes into consideration the recommendations of U.G.C. Curriculum Development Committee and it is meant to be introduced from the academic year 2019-20.

Consolidated List of Courses offered:

Hard Core Courses:

First Semester

1. MTH 401 Algebra – I
2. MTH 402 Linear Algebra – I
3. MTH 403 Real Analysis – I

Second Semester

4. MTH 452 Algebra – II
5. MTH 453 Real Analysis – II
6. MTH 454 Topology

Third Semester

7. MTH 502 Complex Analysis – I
8. MTH 503 Measure and Integration
9. MTH 504 Multivariate Calculus and Geometry

Fourth Semester

10. MTP 551 Project Work
11. MTH 552 Complex Analysis – II

12. MTH 553 Functional Analysis

Soft Core Courses:

1. MTS 404 Numerical Analysis
2. MTS 405 Number Theory
3. MTS 455 Linear Algebra – II
4. MTS 456 Ordinary Differential Equations
5. MTS 505 Advanced Numerical Analysis
6. MTS 506 Commutative Algebra
7. MTS 507 Graph Theory
8. MTS 508 Lattice Theory
9. MTS 509 Fluid Mechanics
10. MTS 510 Theory of Partitions
11. MTS 554 Partial Differential Equations
12. MTS 555 Advanced Topology
13. MTS 556 Advanced Discrete Mathematics
14. MTS 557 Algebraic Number Theory
15. MTS 558 Calculus of Variations and Integral Equations
16. MTS 559 Mathematical Statistics

Open Elective Courses:

1. MTE 451 Discrete Mathematics and Applications
2. MTE 501 Differential Equations and Applications

Labs (Soft core):

1. MTL 406 Lab - 1
2. MTL 457 Lab - 2
3. MTL 511 Lab - 3

A. The following shall be the Courses of study in the four semesters M.Sc. Mathematics Programme (CBCS-PG) from the academic year 2019-2020.

First Semester

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTH 401	Algebra – I	HC	4
MTH 402	Linear Algebra – I	HC	4
MTH 403	Real Analysis – I	HC	4
MTS 404	Numerical Analysis	SC	4
MTS 405	Number Theory	SC	4
MTL 406	Lab - 1	SC	2

Second Semester

In this semester, the course 'MTE 451' is an "Open Elective Course" which is offered only to students of other departments. The other six courses are offered to the students of the department.

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTE 451	Discrete Mathematics and Applications	OE	3
MTH 452	Algebra – II	HC	4
MTH 453	Real Analysis – II	HC	4
MTH 454	Topology	HC	4
MTS 455	Linear Algebra – II	SC	4
MTS 456	Ordinary Differential Equations	SC	4
MTL 457	Lab - 2	SC	2

Third Semester

In this semester, the course 'MTE 501' is an "Open Elective Course" which is offered only to students of other departments. The other courses are offered to the students of the department. The hard core courses MTH 502, MTH 503, MTH 504 and the Lab MTL 511 are compulsory. The student can choose any two soft core courses from MTS 505 to MTS 510. Also, a project work which is compulsory for every student, involves self study to be carried out by the student (on a research problem of current interest or on an advanced topic not covered in the syllabus) under the guidance of a supervisor*. Project work shall be initiated in the third semester itself and the project report (dissertation) shall be submitted at the end of the fourth semester.

*Supervisor from the parent institution or from any other reputed institution/industry.

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTE 501	Differential Equations and Applications	OE	3
MTH 502	Complex Analysis – I	HC	4
MTH 503	Measure and Integration	HC	4
MTH 504	Multivariate Calculus and Geometry	HC	4
MTS 505	Advanced Numerical Analysis	SC	4
MTS 506	Commutative Algebra	SC	4
MTS 507	Graph Theory	SC	4
MTS 508	Lattice Theory	SC	4
MTS 509	Fluid Mechanics	SC	4
MTS 510	Theory of Partitions	SC	4
MTL 511	Lab - 3	SC	2

Fourth Semester

In this semester, the course MTP 551 is a project work which the student has taken up under the guidance of a supervisor in the third semester itself. Each student has to submit a project report (dissertation) at the end of the fourth semester. The hard core courses MTH 552 and MTH 553 are compulsory. The student can choose any two soft core courses from MTS 554 to MTS 559. The soft core course MTS 559 should be taught by the subject expert.

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTP 551	Project Work	Project	4
MTH 552	Complex Analysis – II	HC	4
MTH 553	Functional Analysis	HC	4
MTS 554	Partial Differential Equations	SC	4
MTS 555	Advanced Topology	SC	4
MTS 556	Advanced Discrete Mathematics	SC	4
MTS 557	Algebraic Number Theory	SC	4
MTS 558	Calculus of Variations and Integral Equations	SC	4
MTS 559	Mathematical Statistics	SC	4

B. Scheme of Instruction and Examination
First Semester

Course Code	Instruction hours per week	Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
MTH 401	4	4	3	70	30	100
MTH 402	4	4	3	70	30	100
MTH 403	4	4	3	70	30	100
MTS 404	4	4	3	70	30	100
MTS 405	4	4	3	70	30	100
MTL 406	2	2	3	35	15	50

Second Semester

Course Code	Instruction hours per week	Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
MTE 451	3	3	3	70	30	100
MTH 452	4	4	3	70	30	100
MTH 453	4	4	3	70	30	100
MTH 454	4	4	3	70	30	100
MTS 455	4	4	3	70	30	100
MTS 456	4	4	3	70	30	100
MTL 457	2	2	3	35	15	50

Third Semester

Course Code	Instruction hours per week	Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
MTE 501	3	3	3	70	30	100
MTH 502	4	4	3	70	30	100
MTH 503	4	4	3	70	30	100
MTH 504	4	4	3	70	30	100
MTS 505	4	4	3	70	30	100
MTS 506	4	4	3	70	30	100
MTS 507	4	4	3	70	30	100
MTS 508	4	4	3	70	30	100
MTS 509	4	4	3	70	30	100
MTS 510	4	4	3	70	30	100
MTL 511	2	2	3	35	15	50

Fourth Semester

Course Code	Instruction hours per week	Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
MTP 551	4	4	–	70	30	100
MTH 552	4	4	3	70	30	100
MTH 553	4	4	3	70	30	100
MTS 554	4	4	3	70	30	100
MTS 555	4	4	3	70	30	100
MTS 556	4	4	3	70	30	100
MTS 557	4	4	3	70	30	100
MTS 558	4	4	3	70	30	100
MTS 559	4	4	3	70	30	100

Tutorials: There shall be at least 3 hours of tutorials per week for each course having 4 credits.

Scheme of Evaluation for Internal Assessment Marks:

1. **Theory Course**:

   Each Theory Course shall carry 30 marks for internal assessment based on two tests of 90 minutes duration each.

2. **Project Work:**

   Project Work shall carry 30 marks for internal assessment based on two presentations by the student before a panel of faculty members of the department.

3. **Lab**:

   Each Lab shall carry 15 marks for internal assessment based on two lab tests of 90 minutes duration each.

Pattern of Semester Examination:

1. **Theory Paper:**

   Each question paper for the theory course shall contain EIGHT questions out of which FIVE are to be answered. All questions carry equal marks.

2. **Project Report:**

   The evaluation of a project report is by two examiners as per the regulations.

3. **Lab Exam:**

   Each Lab exam question paper shall contain TWO questions on lab programmes which are to be executed.

C. Syllabi of Each Semester

I Semester

MTH 401	Algebra – I	4 Credits (48 hours)

Unit I - Groups and Subgroups:
Binary operations, Isomorphic binary operations, Groups, Subgroups, Cyclic groups, Generating sets and Cayley digraphs, Groups of permutations, Orbits, Cycles and alternating groups, Cosets and Lagrange's theorem. **(12 Hours)**

Unit II - Product Groups, Homomorphism and Quotient Groups:
Direct products and finitely generated abelian groups, Homomorphisms, Factor groups, Factor group computations and simple groups, Isomorphism theorems. Series of groups. **(12 Hours)**

Unit III - Advanced Group Theory:
Symmetry of plane figures, Isometries, Isometries of the plane, Finite groups of orthogonal operators on the plane. Group actions on a set, Applications of group actions to counting, Cayley's theorem, The class equation, p-Groups, Conjugation in the symmetric group, Normalizers, The Sylow theorems, The groups of order 12. **(18 Hours)**

Unit IV - Rings and Fields:
Definitions of rings, subrings, integral domains, fields and their basic properties, Homomorphisms and Factor Rings, Prime and Maximal Ideals. Fields of quotients of an integral domain, Rings of Polynomials. **(6 Hours)**

References

[1] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[2] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[5] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[6] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[7] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[8] Serge Lang, *Algebra*, 3rd Ed., Springer, 2005.

MTH 402	Linear Algebra – I	4 Credits (48 hours)

Unit I - Matrix Operations:
Recapitulation of the basic operations, Block multiplication, Matrix units, Row reduction, The matrix transpose, Permutation matrices, Determinants, Other formulas for Determinant, The Cofactor matrix. **(12 Hours)**

Unit II - Vector Spaces:
Subspaces of $\mathbb{R}^n$, Fields, Vector Spaces, Bases and dimension. Computing with bases, Direct sums, Infinite Dimensional spaces. **(12 Hours)**

Unit III - Linear Operators:
The dimension formula, The matrix of a linear transformation, Linear Operators, Eigenvectors, The characteristic polynomial, Triangular and Diagonal forms. Jordan form. **(18 Hours)**

Unit IV - Applications of Linear Operators:
Orthogonal matrices and Rotations, Cayley-Hamilton Theorem, The matrix exponential. **(6 Hours)**

References

[1] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[2] Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra*, 4th Ed., Prentice Hall of India, 2014.

[3] K. Hoffmann and R. Kunz, *Linear Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[4] Serge Lang, *Linear Algebra*, Addison Wesley, London, 1970.

[5] Larry Smith, *Linear Algebra*, 3rd Ed., Springer Verlag, 1998.

[6] Gilbert Strang, *Linear Algebra and its Applications*, 4th Ed., Cengage Learning, 2006.

[7] S. Kumaresan, *Linear Algebra - A Geometric Approach*, PHI, 2003.

MTH 403	Real Analysis – I	4 Credits (48 hours)

Unit I
The Real and Complex Number System: Introduction, Ordered sets, Fields, The real field, The extended real number system, The complex field, Euclidean spaces, Inequalities.
Basic Topology: Finite, countable and uncountable sets, Metric spaces, Compact sets, Perfect sets, Connected sets. **(16 Hours)**

Unit II - Numerical Sequences and Series:
Convergent sequences, Subsequences, Cauchy sequences, Upper and lower limits, Some special sequences, Series, Series of non-negative terms, The number e, The root and ratio tests, Power series, Summation by parts, Absolute convergence, Addition and multiplication of series. Rearrangements. **(12 Hours)**

Unit III - Continuity:
Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and limits at infinity. **(12 Hours)**

Unit IV - Differentiation:
The derivative of a real function, Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher order, Taylor's theorems, Differentiation of vector valued functions. **(8 Hours)**

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[4] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[5] R. R. Goldberg , *Methods of Real Analysis*, 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[6] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[7] Russel A. Gordon, *Real Analysis - A first Course*, 2nd Ed., Pearson, 2011.

MTS 404	Numerical Analysis	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Unit I - Transcendental and Polynomial Equations:
Introduction, The bisection method, Iteration methods based on first degree equation, Iteration methods based on second degree equation, Rate of convergence, Rate of convergence of Secant and Newton-Raphson method. Iteration methods - First order method, Second order method, Higher order methods. Polynomial equations, Descartes' Rule of Signs, The Birge-Vieta method. **(12 Hours)**

Unit II - System of Linear Equations and Eigenvalue problems:
Introduction, Direct methods - Gauss elimination method, Gauss-Jordan method, Triangularization method, Cholesky method. Iteration methods - Jacobi iteration method, Gauss-Seidel iteration method, Convergence analysis, Eigenvalues and eigenvectors. The power method. **(12 Hours)**

Unit III - Interpolation and Approximation:
Introduction, Lagrange and Newton interpolations, Linear and Higher order interpolation, Finite difference operators, Interpolating polynomials using finite differences, Hermite interpolation, Approximations. **(12 Hours)**

Unit IV
Numerical Differentiation: Introduction, Methods based on Interpolation, Methods based on finite differences, Methods based on undetermined coefficients, Extrapolation methods.
Numerical Integration: Methods based on Interpolation, Newton-Cotes methods, Composite Integration Methods. **(12 Hours)**

References

[1] M. K. Jain, S. R. K. Iyengar, R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] A. Ralston and P. Rabinowitz, *A First Course in Numerical Analysis*, 2nd Ed., McGraw - Hill, New York, 1978.

[4] K. Atkinson, *Elementary Numerical Analysis*, 2nd Ed., John Wiley and Sons, Inc., 1994.

[5] P. Henrici, *Elements of Numerical Analysis*, John Wiley and Sons, Inc., New York, 1964.

MTS 405	Number Theory	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Unit I
Divisibility and Primes: Recapitulation of Division algorithm, Euclid's algorithm, Least Common Multiples, Linear Diophantine equations. Prime numbers and Prime-power

factorisations, Distribution of primes, Fermat and Mersenne primes, Primality testing and factorization.

Arithmetical Functions: The Möbius function and its properties, Euler function, examples and properties, The Dirichlet product of arithmetical functions, Dirichlet inverses and the Möbius inversion formula. **(12 Hours)**

Unit II - Congruences:

Recapitulation of basic properties of congruences, Residue classes and complete residue systems, Linear congruences. Reduced residue systems and the Euler-Fermat theorem, Polynomial congruences modulo p and Langrange's theorem, Simultaneous linear congruences, Simultaneous non-linear congruences, An extension of Chinese Remainder Theorem, Solving congruences modulo prime powers. **(12 Hours)**

Unit III - Quadratic Residues and Quadratic Reciprocity Law:

Quadratic residues, Legendre's symbol and its properties, Euler's criterion, Gauss lemma, The quadratic reciprocity law and its applications, The Jacobi symbol, Applications to Diophantine equations. **(12 Hours)**

Unit IV - Sums of squares, Fermat's last theorem and Continued fractions:

Sums of two squares, Sums of four squares, The Pythagoras theorem, Pythagorean triples and their classification, Fermat's Last Theorem (Case $n = 4$).

Recapitulation of Finite continued fractions, Infinite continued fractions, Representation of irrational numbers, Periodic continued fractions and quadratic irrationals. Solution of Pell's equation by continued fractions. **(12 Hours)**

References

[1] G. A. Jones and J. M. Jones, *Elementary Number Theory*, Springer UTM, 2007.

[2] Tom M. Apostol, *Introduction to Analytic Number Theory*, Springer, 1989.

[3] David M. Burton, *Elementary Number Theory*, 7th Ed., McGraw-Hill, 2010.

[4] Niven, H.S. Zuckerman & H.L. Montgomery, *Introduction to the Theory of Numbers*, Wiley, 2000.

[5] H. Davenport, *The Higher Arithmetic*, Cambridge University Press, 2008.

MTL 406	Lab - 1	2 Credits (2 hours lab /week)

Practicals for I Semester

Mathematics practicals with Free and Open Source Software (FOSS) tools for computer programs

1) Program to accept three integers and print the largest among them and program to check whether a given integer is even or odd and also positive or negative.

2) Program to find roots of a quadratic equation.

3) Program to perform arithmetic operations using switch case.

4) Program to convert binary number to decimal number and decimal number to binary number.

5) Program to calculate factorial of a number and program to print Fibonacci numbers.

6) Program to search an element in the array.

7) Program to arrange a set of given integers in an ascending order and print them.

8) Program to find row sum and column sum of a matrix.

9) Program to find the Transpose, Trace and Norm of a matrix.

10) Program to find sum, difference and product of two matrices.

11) Program to test whether a given integer is a prime and program to generate prime numbers between two give numbers.

12) Program to find the Armstrong Number between two given numbers and program to test whether a given number is Palindrome or not.

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

II Semester

MTE 454	Discrete Mathematics and Applications	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to XII/PU.

Unit I - Number Theory and Cryptography:
Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography. **(8 Hours)**

Unit II - Counting Techniques:
The Basics of Counting, The Pigeon-hole Principle, Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Recurrence Relations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Generating Functions. Principle of Inclusion-Exclusion, Applications of Inclusion-Exclusion. **(12 Hours)**

Unit III - Order Relations and Structures:
Product Sets and Partitions, Relations, Properties of Relations, Equivalence Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Boolean Functions as Boolean Polynomials. **(8 Hours)**

Unit IV - Groups and Coding Theory:
Binary Operations Revisited, Semigroups, Products and Quotients of Semigroups, Groups, Products and Quotients of Groups, Coding of Binary Information and Error Detection, Decoding and Error Correction. **(8 Hours)**

References

[1] Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Ed., Tata Mc-Graw-Hill, 2012.

[2] Bernard Kolman, Robert C. Busby, Sharon Cutler Ross, *Discrete Mathematical Structures*, 3rd Ed., Prentice Hall, 1996.

[3] Grimaldi R, *Discrete and Combinatorial Mathematics*, 5th Ed., Pearson, Addison Wesley, 2004.

MTH 452	Algebra – II	4 Credits (48 hours)

Unit I - Factoring:
Unique factorization domains, Euclidean domains, Content of polynomials, Primitive polynomials, Gauss lemma, Unique factorization in $R[x]$, where R is a U.F.D., Irreducibility test mod p, Eisenstein's criterion, Gauss primes. **(16 Hours)**

Unit II - Fields:
Algebraic and transcendental elements, The degree of a field extension, Finding the irreducible polynomial, Ruler and compass constructions, Isomorphism of field extensions, Adjoining roots, Splitting fields, Finite fields, Primitive elements, Algebraically closed fields, The fundamental theorem of algebra. **(20 Hours)**

Unit III - Galois Theory:
Automorphisms and Fields, Separable Extensions, Galois Theory, Illustrations of Galois Theory, Cyclotomic Extensions, Insolvability of the Quintic. **(12 Hours)**

References

[1] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[5] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[6] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[7] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[8] Serge Lang, *Algebra*, 3rd Ed., Springer, 2005.

MTH 453	Real Analysis – II	4 Credits (48 hours)

Unit I
The Riemann-Stieltjes Integral:
Definition and existence of integrals, Properties of integral, Integration and differentiation, Integration of vector-valued functions, Rectifiable curves.

Improper Integrals: Definition, Criteria for convergence, Interchanging derivatives and integrals. **(20 Hours)**

Unit II - Sequences and Series of Functions:
Discussion of main problem, Uniform convergence, uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Equicontinuous families of functions, The Stone-Weierstrass theorem. **(16 Hours)**

Unit III - Functions of Several Variables:
Differentiation, The contraction principle, The inverse function theorem, The implicit function theorem. **(12 Hours)**

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] Serge Lang, *Analysis I*, Addison Wesley Publishing Company, 1968.

[4] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[5] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[6] R. R. Goldberg , *Methods of Real Analysis*, 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[7] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[8] Russel A. Gordon, *Real Analysis - A first Course*, 2nd Ed., Pearson, 2011.

MTH 454	Topology	4 Credits (48 hours)

Unit I - Topological Spaces:
The definition and some examples, Elementary concepts, Open bases and open subbases, Weak topologies, The function algebras $\mathcal{C}(X, \mathbb{R})$ and $\mathcal{C}(X, \mathbb{C})$. **(15 Hours)**

Unit II - Compactness:
Compact Spaces, Product spaces, Tychonoff's theorem. **(10 Hours)**

Unit III - Separation:
T_1-Spaces and Hausdorff spaces, Completely regular spaces and Normal spaces, Urysohn's lemma and Tietze extension theorem, The Urysohn imbedding theorem. **(13 Hours)**

Unit IV - Connectedness:
Connected spaces, The components of a space, Totally disconnected spaces, Locally connected spaces. **(10 Hours)**

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[2] J. R. Munkres, *Topology*, 2nd Ed., Pearson Education, Inc, 2000.

[3] S. Willard, *General Topology*, Addison Wesley, New York, 1968.

[4] J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.

[5] J. L. Kelley, *General Topology*, Van Nostrand Reinhold Co., New York, 1955.

MTS 455	Linear Algebra – II	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 402 (Linear Algebra – I).

Unit I - Bilinear Forms:
Bilinear forms, Symmetric forms, Hermitian forms, Orthogonality, Orthogonal Projection, Euclidean and Hermitian spaces, The spectral theorem, Skew symmetric forms, Summary of results in matrix notation. **(24 Hours)**

Unit II - Linear Algebra in a Ring:
Modules, Free modules, Diagonalizing Integer Matrices, Submodule of free modules, Generators and Relations, Noetherian Rings, The structure theorem for abelian groups, Application to linear operators. **(24 Hours)**

References

[1] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[2] Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra*, 4th Ed., Prentice Hall of India, 2014.

[3] K. Hoffmann and R. Kunz, *Linear Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[4] Serge Lang, *Linear Algebra*, Addison Wesley, London, 1970.

[5] Larry Smith, *Linear Algebra*, 3rd Ed., Springer Verlag, 1998.

[6] Gilbert Strang, *Linear Algebra and its Applications*, 4th Ed., Cengage Learning, 2006.

[7] S. Kumaresan, *Linear Algebra - A Geometric Approach*, PHI, 2003.

MTS 456	Ordinary Differential Equations	4 Credits (48 hours)

Prerequisite: Knowledge of syllabi prescribed for the courses MTH 402 (Linear Algebra – I) and MTH 403 (Real Analysis – I).

Unit I - Linear Differential Equations of Higher Order:
Linear dependence and the Wronskian, Basic theory for linear equations, Method of variation of parameters, Reduction of n^{th} order linear homogeneous equation, Homogeneous and

non-homogeneous equations with constant coefficients. **(12 Hours)**

Unit II - Solutions in Power Series:
Second order linear equations with ordinary points, Legendre equation and Legendre polynomials, Second order equations with regular singular points, Bessel equation. **(18 Hours)**

Unit III - Systems of Linear Differential Equations:
Systems of first order equations, Existence and uniqueness theorem. The fundamental matrix, Non-homogeneous linear systems, Linear systems with periodic coefficients. **(10 Hours)**

Unit IV - Existence and Uniqueness of solutions :
Equations of the form $x' = f(t, x)$, Method of successive approximation, Lipschitz condition, Picard's theorem, Non uniqueness of solutions, Continuation of solutions. **(8 Hours)**

References

[1] S. G. Deo and V. Raghavendra, *Ordinary Differential Equations and Stability Theory*, Tata McGraw Hill, 1980.

[2] A. Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall of India, 2013.

[3] A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Krieger, 1984.

[4] M. W. Hirsh and S. Smale, *Differential Equations, Dynamical Systems and Linear Algebra*, Academic Press, New York, 1974.

[5] V. I. Arnold, *Ordinary Differential Equations*, MIT Press, Cambridge, 1981.

[6] Shepley L. Ross, *Differential Equations*, Wiley, 2004.

MTL 457	Lab - 2	2 Credits (2 hours lab /week)

Practicals for II Semester

Mathematics practicals with Free and Open Source Software (FOSS) tools for computer programs

1) Program to find solution to a system of linear equations by matrix inversion method (check for all conditions on input matrix).

2) Program to find solution to a system of linear equations by Cramers rule (check for all conditions on input matrix).

3) Program to find area of one of the geometric figures (circle, triangle, rectangle and square) using switch statements.

4) Program to implement Newton Gregory Forward Difference method.

5) Program to implement Lagrange interpolation polynomial.

6) Program to find the value of a function by using Hermite interpolation method.

7) Program to plot a neat labeled graph of elementary functions on the same plane.

8) Program to obtain the graph of plane curves - cycloid and astroid in separate figure on a single run.

9) Program to obtain a neat labeled graph of space curves - elliptical helix and circular helix in separate figure on a single run.

10) Program to obtain a neat labeled graph of surfaces - elliptic paraboloid and hyperbolic paraboloid in separate figure on a single run.

11) Program to animate the plotted curves.

12) Program to find extreme values of functions of a single variable.

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

III Semester

MTE 501	Differential Equations and Applications	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to XII/PU.

Unit I
Recapitulation of methods of solutions of first order differential equations, Applications of First Order Ordinary Differential Equations - Simple problems of dynamics – falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay. **(10 Hours)**

Unit II
Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms. **(10 Hours)**

Unit III
Power series solutions of Second Order Linear Differential Equations, their mathematical properties. Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials, Chebyshev polynomials, Hermite polynomials and Laguerre polynomials.
(16 Hours)

References

[1] G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill, New Delhi, 1991.

[2] D. Rainville and P. Bedient, *Elementary course on Ordinary Differential Equations*, Macmillan, New York, 1972.

[3] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol. I, Tata McGraw Hill, New Delhi, 1975.

MTH 502	Complex Analysis – I	4 Credits (48 hours)

Unit I - Complex numbers and Complex Functions:
Recapitulation of the algebra of complex numbers - Arithmetic operations, Square roots, Conjugation, Absolute value, Inequalities.
The geometric representation of complex numbers - Geometric addition and multiplication, The binomial equation, Analytic geometry, The spherical representation.
Introduction to the concept of analytic function - Limits and continuity, Analytic functions, Polynomials, Rational functions.
Elementary theory of power series - Sequences, Series, Uniform convergence, Power series, Abel's limit theorem. The exponential and trigonometric functions - The exponential, The trigonometric functions, The periodicity, The logarithm. **(18 Hours)**

Unit II - Analytic Functions as Mappings, Complex Integration :
Elementary Point set Topology - All topological properties to be reviewed, with an emphasis on Connectedness, and compactness.
Conformality - Arcs and closed curves, Analytic functions in regions, Conformal mapping, Length and area.
Linear transformation - The linear group. The cross ratio, Symmetry.
Fundamental theorems - Line integrals, Rectifiable arcs, Line integrals as function of arcs, Cauchy's theorem for a rectangle, Cauchy's theorem for a disk.
Cauchy's Integral Formula - The index of a point with respect to a closed curve, The integral formula, Higher derivatives. **(16 Hours)**

Unit III - Local Properties of Analytical Functions:
Removable singularities, Taylor's theorem, Zeros and poles, The local mapping, The maximum principle.
The General Form of Cauchy's Theorem - Chains and cycles, Simple connectivity, Homology, The general statement of Cauchy's theorem - Cauchy's theorem. Locally exact differentials, Multiply connected regions. **(14 Hours)**

References

[1] Lars V. Ahlfors, *Complex Analysis*, 3rd Ed., McGraw Hill, 1979.

[2] B. R. Ash, *Complex Variables*, 2nd Ed., Dover Publications, 2007.

[3] R. V. Churchill, J. W. Brown and R. F. Verlag, *Complex Variables and Applications*, 8th Ed., Mc Graw Hill, 2009.

[4] J. B. Conway, *Functions of one Variable*, Narosa, New Delhi, 1996.

[5] S. Ponnuswamy and H. Silverman, *Complex Variables with Applications*, Birkäuser, 2006.

MTH 503	Measure and Integration	4 Credits (48 hours)

Unit I
Algebras of sets - Borel sets. Outer measure, Measurable sets and Lebesgue measure. Example of a non-measurable set. Measurable functions. **(12 Hours)**

Unit II

The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of a nonnegative function, The general Lebesgue integral. **(12 Hours)**

Unit III

Differentiation and Integration, Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity. **(12 Hours)**

Unit IV

Measure and outer measure, The extension theorem of Caratheodary, The product measures, The Fubini theorem. **(12 Hours)**

References

[1] H. L. Royden, *Real Analysis*, 3rd Ed., Prentice - Hall, 2003.

[2] G. D. Barra, *Introduction to Measure Theory*, Van Nostrand Reinhold Company Ltd., 1974.

[3] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., Tata McGraw Hill Publishing Company, 1987.

[4] P. R. Halmos, *Measure Theory*, Springer Verlag, 1974.

[5] F. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer Verlag, 1965.

[6] Inder K. Rana, *An Introduction to Measure and Integration*, 2nd Ed., Narosa Publishing House, 1997.

MTH 504	Multivariate Calculus and Geometry	4 Credits (48 hours)

Unit I

Introduction to differentiable functions, Level sets and tangent spaces, Lagrange multipliers, Maxima and minima on open sets. **(12 Hours)**

Unit II

Curves in $\mathbb{R}^3$, Line Integrals, The Frenet-Serret equations, Geometry of curves in $\mathbb{R}^3$. **(12 Hours)**

Unit III

Double integration - Green's theorem. Parametrised surfaces in $\mathbb{R}^3$, Surface area, Surface integrals, Stoke's theorem, Triple integrals, The divergence theorem. **(16 Hours)**

Unit IV

The geometry of surfaces in $\mathbb{R}^3$, Gaussian Curvature, Geodesic. **(8 Hours)**

References

[1] Sean Dineen, *Multivariate Calculus and Geometry*, 3rd Ed., Springer Undergraduate Mathematics Series, 2014.

[2] Andrew Pressly, *Elementary Differential Geometry*, 2nd Ed., Springer Undergraduate Mathematics Series, 2010.

[3] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, New York, 1976.

[4] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Undergraduate Texts in Mathematics, Springer Verlag, 1994.

[5] W. Klingenberg, *A course in Differential Geometry*, Springer Verlag, 1983.

MTS 505	Advanced Numerical Analysis	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 404 (Numerical Analysis).

Unit I - Numerical Integration:
Recapitulation of the methods based on interpolation, Methods based on undetermined coefficients. Romberg integration, Gauss-Legendre integration methods, Gauss-Chebyshev integration methods, Gauss-Laguerre integration methods, Gauss-Hermite integration methods. Double integration, Trapezoidal rule, Simpson's rule. **(15 Hours)**

Unit II - Ordinary Differential Equations:
Introduction, Numerical methods, Euler method, Backward Euler method, Mid-point method, Single step methods, Taylor series method, Runge-Kutta methods, Multistep methods, Determination of a_j and b_j, Predictor-corrector methods, Boundary value problems, Finite difference methods, Trapezoidal, Dahlquist and Numerov methods.
(15 Hours)

Unit III - Systems of Linear Differential Equations:
Introduction, Difference methods, Parabolic equations in one space dimension, Schmidt formula, Du Fort-Frankel scheme, Crank-Nicolson and Crandall schemes, Solution of hyperbolic equation in one dimension by explicit schemes, The CFL condition, Elliptic equations, Dirichlet problem, Neumann problem, Mixed problem. **(18 Hours)**

References

[1] M. K. Jain, S. R. K. Iyengar, P. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] M. K. Jain, *Numerical Solution of Differential Equations*, 2nd Ed., New Age International (P) Ltd., New Delhi, 1984.

[4] A. R. Mitchell, *Computational Methods in Partial Differential Equations*, John Wiley and Sons, Inc., 1969.

MTS 506	Commutative Algebra	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the courses MTH 401 (Algebra – I) and MTH 452 (Algebra – II).

Unit I - Rings and Ideals:
Zero divisors, Nilpotent elements, Units, Prime ideals and maximal ideals, Nilradical and Jacobson radical, Operations on ideals, Extensions and contraction of ideals. **(16 Hours)**

Unit II - Modules:
Recapitulation of Operations on submodules, Isomorphism theorems. Direct sum and product, Finitely generated modules, Nakayama's lemma, Exact sequences (omit tensor products and related results). **(12 Hours)**

Unit III - Rings and Modules of Fractions:
Local properties, Extended and contracted ideals in rings of fractions. **(10 Hours)**

Unit IV - Primary Decomposition, Integral Dependence and Chain Conditions:
First and second uniqueness theorems, Integral dependence, The going-up theorem, Integrally closed integral domains, The going-down theorem, Noetherian rings and modules, Primary decomposition in Noetherian rings. **(10 Hours)**

References

[1] M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Indian Ed., Lavant Books, 2007.

[2] N. Bourbaki, *Commutative Algebra*, American Mathematical Society, 1972.

[3] N. S. Gopalkrishnan, *Commutative Algebra*, 2nd Ed., University Press, 2015.

[4] G. Northcott, *Lesson on Rings, Modules and Multiplicities*, Cambridge University Press, 2008.

[5] O. Zariski and P. Samuel, *Commutative Algebra Vol I, II*, Graduate Texts in Mathematics, Springer Verlag, 1976.

MTS 507	Graph Theory	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Unit I - Graphs:
Varieties of graphs, Walks and connectedness, Degrees, The problem of Ramsey, Extremal graphs, Intersection graphs, Operations on graphs. **(10 Hours)**

Unit II - Blocks, Trees and Connectivity:
Cut points, Bridges, Blocks, Block graphs and Cut point graphs, Characterization of trees, Centers and centroids, Block-Cutpoint trees, Independent cycles and cocycles. Connectivity

and line-connectivity, Graphical variations of Menger's theorem. **(15 Hours)**

Unit III - Traversability and Planarity:
Eulerian graphs, Hamiltonian graphs. Plane and planar graphs Outerplanar graphs. **(15 Hours)**

Unit IV
Colorability: The chromatic number, The Five Color Theorem, The chromatic polynomial.
Matrices: The adjacency matrix, The incidence matrix and The cycle matrix, Matrix-Tree Theorem. **(8 Hours)**

References

[1] F. Harary, *Graph Theory*, Addison-Wesley Series in Mathematics, 1969.

[2] Narsingh Deo, *Graph Theory with Applications to Engineering and Computer Science*, Prentice Hall of India, 1987.

[3] Bela Bollabas, *Modern Graph theory*, Springer, 1998.

[4] R. Balakrishnan and K. Ranganathan, *A textbook of Graph Theory*, Springer-Verlag, 2000.

[5] Douglass B. West, *Introduction to Graph Theory*, Prentice Hall of India, New Delhi, 1996.

[6] O. Ore, *Theory of Graphs*, American Mathematical Society, Providence, Rhode Island, 1967.

MTS 508	Lattice Theory	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Unit I - Partially Ordered Sets:
Partially ordered sets (or Posets), Diagrams, Lower and upper bounds, Order homomorphism and order isomorphism, Special subsets of a poset. Axiom of choice (Statement only). Zorn's lemma and Hausdorff's maximal chain principle, and proof of the equivalence of these two statements. Length of a poset, The minimum and maximum conditions, Duality principle for posets. **(12 Hours)**

Unit II - Lattices in General:
A lattice as a poset and as an algebra, Diagrams of lattices, Duality principle for lattices, Semilattices, Sublattices, Ideals and prime ideals of lattices, Ideal generated by a nonempty subset of a lattice and its description, The ideal lattice and the augmented ideal lattice of a lattice, Bound elements, atoms and dual atoms in a lattice, Atomic lattices, complemented, relatively complemented and sectionally complemented lattices, Homomorphisms, congruence relations and quotient lattices of lattices, The homomorphism theorem. **(12 Hours)**

Unit III - Complete Lattices:
Complete lattices, fixed point property. Compact elements and compactly generated lattices.

(6 Hours)

Unit IV - Distributive and Modular Lattices:
Distributive, Modular lattices, Characterizations of modular and distributive lattices in terms of sublattices, The isomorphism theorem of modular lattices, The prime ideal theorem for distributive lattices. **(10 Hours)**

Unit V - Complemented Modular Lattices and Boolean Algebras:
Complemented modular lattices and bounded relatively complemented lattices. Distributivity of a uniquely complemented relatively complemented lattice, Boolean algebras, De Morgan formulae, Boolean algebras and Boolean rings, Distributive lattices and rings of sets, Boolean algebras and fields of sets. **(8 Hours)**

References

[1] G. Szasz, *Introduction to Lattice Theory*, Academic Press, N.Y., 1963.

[2] G. Gratzer, *General Lattice Theory*, Birkhäuser Verlag, Basel, 1978.

[3] P. Crawley and R.P, Dilworth, *Algebraic Theory of Lattices*, Prentice - Hall Inc., N. J., 1973.

[4] G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, Volume 25, 1995.

[5] L. A. Skornjakov, *Elements of Lattice Theory*, Hindustan Publishing Corporation, 1977.

MTS 509	Fluid Mechanics	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Unit I - Motion of Inviscid Fluids:
Recapitulation of equation of motion and standard results, Vortex motion-Helmholtz vorticity equation, Permanence of vorticity and circulation, Kelvin's minimum energy theorem – Impulsive motion, Dimensional analysis, Nondimensional numbers. **(8 Hours)**

Unit II - Two Dimensional Flows of Inviscid Fluids:
Meaning of two-dimensional flow, Stream function, Complex potential, Line sources and sinks, Line doublets and vortices, Images, Milne-Thomson circle theorem and applications, Blasius theorem and applications. **(10 Hours)**

Unit III - Motion of Viscous Fluids:
Stress tensor, Navier-Stokes equation, Energy equation, Simple exact solutions of Navier-Stokes equation: (i) Plane Poiseuille and Hagen- Poiseuille flows (ii) Generalized plane Couette flow (iii) Steady flow between two rotating concentric circular cylinders (iv) Stokes's first and second problems (vi) Slow and steady flow past a rigid sphere and cylinder. Diffusion of vorticity, Energy dissipation due to viscosity. Boundary layer concept, Derivation of Prandtl

boundary layer equations, Blasius solution – Karman's integral equation. **(14 Hours)**

Unit IV - Gas Dynamics:
Compressible fluid flows, Standard forms of equations of state, Speed of sound in gas, Equations of motion of non-viscous and viscous compressible flows. Subsonic, sonic and supersonic flows, Isentropic flows, Gas dynamical equations. **(8 Hours)**

Unit V - Turbulent Flow:
Introduction, Transition from laminar to turbulent flow, Homogeneous turbulence, Isotropic turbulence, Spatial, time and ensemble averages, Basic properties of averages, Reynolds averaging procedure, Derivation of turbulent equations using Reynolds averaging procedure with gradient-diffusion i.e., K-model for closure. **(8 hours)**

References

[1] F. Chorlton, *Text book of Fluid Dynamics*, Van Nostrand, 1967.

[2] L. M. Milne –Thomson, *Theoretical Hydrodynamics*, 4th Ed., Macmillan, 1960.

[3] S. W. Yuan, *Foundations of Fluid Mechanics*, Prentice Hall, 1976.

[4] Z. U. A.Warsi, *Fluid Dynamics*, 2nd Ed., CRC Press, 1999.

[5] B.K. Shivamoggi, *Theoretical Fluid Dynamics*, John Wiley and Sons, 1998.

[6] Stephen B. Pope, *Turbulent Flows*, CambridgeUniversity Press, 2000.

[7] C.S. Yih, *Fluid Mechanics*, McGraw-Hill, 1969.

[8] E.L. Cussier, *Difussion Mass Fluid Systems*, 2nd Ed., Cambridge University Press, 2006.

MTS 510	Theory of Partitions	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 405 (Number Theory).

Unit I
Partitions - partitions of numbers, the generating function of $p(n)$, other generating functions, two theorems of Euler, Jacobi's triple product identity and its applications. **(12 Hours)**

Unit II
$1\psi1$ - summation formula and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof. **(12 Hours)**

Unit III
Congruence properties of partition function, the Rogers - Ramanujan Identities. **(12 Hours)**

Unit IV
Elementary series - product identities, Euler's, Gauss's, Heine's, Jacobi's identities. Restricted Partitions - Gaussian, Frobinius partitions. **(12 Hours)**

References

[1] G. H. Hardy and E. M. Wright, *An Introduction to Theory of Numbers*, 5th Ed., Oxford University Press, 1979.

[2] I. Niven, H. S. Zuckerman and H. L. Montgomery, *An Introduction to the Theory of Numbers*, 5th Ed., New York, John Wiley and Sons, Inc., 2004.

[3] Bruce C. Berndt, *Ramanujan's Note Books Volumes-1 to 5.*

[4] G. E. Andrews, *The Theory of Partitions*, Addison Wesley, 1976.

[5] A. K. Agarwal, Padmavathamma, M. V. Subbarao, *Partition Theory*, Atma Ram & Sons, Chandigarh, 2005.

MTL 511	Lab - 3	2 Credits (2 hours lab /week)

Practicals for III Semester

Mathematics practicals with Free and Open Source Software (FOSS) tools for computer programs

1) Program to implement Least square approximation Method.

2) Program to find a real root of a polynomial using fixed point iterative Method.

3) Program to find a real root of a polynomial using Newton Raphson Method.

4) Program to find a real root of a polynomial using Secant Method.

5) Program to solve a system of equations using Gauss Elimination Method and Gauss Jordan Method.

6) Program to find the solution of a system of equations using using Jacobi Iterative Method/Gauss Seidal Method.

7) Program to find the largest eigenvalue and eigenvector of a matrix by using Power Method.

8) Program to find the smallest eigenvalue and eigenvector of a matrix using inverse power method.

9) Program to find the tri-diagonal matrix using House holder's method.

10) Program to evaluate the given integral using Trapezoidal rule/ Simpson's 1/3 rule/Simpson's 3/8 rule.

11) Program to find the approximate solution of a differential equation with initial condition by Picard's method of successive approximation

12) Program to solve an initial value problem using Euler's Method/ Euler's Modified Method.

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

IV Semester

MTH 552	Complex Analysis – II	4 Credits (48 hours)

Unit I

The Calculus of Residues: The Residue theorem, The argument principle, Evaluation of definite integrals.

Harmonic Functions: Definition and basic properties, The mean value property, Poisson's formula, Schwarz's theorem, The reflection principle. **(12 Hours)**

Unit II - Series and Product Developments:

Power series expansions - Weierstrass's theorem, The Taylor series, The Laurent series. **(12 Hours)**

Unit III - Partial Fractions and Factorization:

Partial fractions, Infinite products, Canonical products, The Gamma function, Jensen's formula, Product development of Riemann Zeta function. **(12 Hours)**

Unit IV

Elliptic Functions: Simply periodic functions - Representation by exponentials, The Fourier development, Function of finite order.

Doubly Periodic Functions: The period module, Unimodular transformation, General properties of elliptic functions. The Weierstrass function. **(12 Hours)**

References

[1] Lars V. Ahlfors, *Complex Analysis*, 3rd Ed., McGraw Hill, 1979.

[2] B. R. Ash, *Complex Variables*, 2nd Ed., Dover Publications, 2007.

[3] R. V. Churchill, J. W. Brown and R. F. Verlag, *Complex Variables and Applications*, 8th Ed., McGraw Hill, 2009.

[4] J. B. Conway, *Functions of one Variable*, Narosa, New Delhi, 1996.

[5] S. Ponnuswamy and H. Silverman, *Complex Variables with Applications*, Birkäuser, 2006.

MTH 553	Functional Analysis	4 Credits (48 hours)

Unit I

Review of metric spaces: Convergence, Completeness and Baire's theorem.

Banach spaces: Definition and some examples, Continuous linear transformations, The Hahn Banach theorem, The natural embedding of N in N^{**}, The open mapping theorem, Uniform boundedness principle. **(26 Hours)**

Unit II- Hilbert spaces:

Definition and examples, Orthogonal complements, Orthonormal sets, The conjugate of a Hilbert space, The adjoint operator, Self-adjoint operators, Normal and unitary operators, Projections, Finite dimensional spectral theorem. **(22 Hours)**

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2004.

[2] A. E. Taylor, David Lay, *Introduction to Functional Analysis*, John Wiley and Sons, 1980.

[3] Ward Cheney, *Analysis for Applied Mathematics*, Graduate Texts in Mathematics, Springer, 2001.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., McGraw Hill, 1986.

[5] M. Thamban Nair, *Functional Analysis - A First Course*, Prentice-Hall, 2002.

MTS 554	Partial Differential Equations	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Unit I

Ordinary differential equations in more than two variables: Recapitulation of Methods of solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, Pfaffian differential forms and Pfaffian differential equations and solutions. Orthogonal trajectories of a system of curves on a surface.

First order partial differential equations: Origin of first order partial differential equations, The Cauchy problem for first order equations, Linear equations of first order, Integral surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces, Nonlinear equations of first order, Cauchy's method of characteristics, Charpit's method, Special types of first order equations. **(24 Hours)**

Unit II

Higher Order Partial Differential Equations: Linear partial differential equations with constant coefficients, Classification of second order PDE, Canonical forms, Adjoint operators, Riemann's method.

Elliptic Differential Equations: Dirichlet problem for a rectangle, Neumann problem for a rectangle, interior and exterior Dirichlet problem for a circle, Interior Neumann problem for a circle. Solution of Laplace equation in Cylindrical and Spherical coordinates.

Parabolic Differential Equations: Occurrence of the diffusion equation, Elementary solutions of the diffusion equation, Dirac Delta function, Separation of variables, Solution of diffusion equation in Cylindrical and spherical coordinates.

Hyperbolic Differential Equations: Solution of one dimensional equation by canonical reduction, Initial value problem - D'Alembert's solution, Vibrating string - variable separation method, Forced vibrations, Boundary and initial value problems for two dimensional wave equation, Uniqueness of the solution for the wave equation, Duhamel's principle. **(24 Hours)**

References

[1] Ian Sneddon, *Elements of Partial Differential Equations*, International student Ed., McGraw Hill, 1957.

[2] K. Sankara Rao, *Introduction to Partial Differential Equations*, Prentice-Hall of India, 1995.

[3] F. John, *Partial Differential Equations*, Springer Verlag, New York, 1971.

[4] P. Garabedian, *Partial Differential Equations*, Wiley, New York, 1964.

MTS 555	Advanced Topology	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 454 (Topology).

Unit I - Preliminaries :
Order relations and dictionary order relations, Well ordering theorem, The minimal uncountable well ordered set S_Ω and its basic properties. The order topology and the ordered square I_0^2, the least upper bound property of I_0^2. Box and product topologies on arbitrary products of spaces and continuity of a function from a space into these products. Compact sets in ordered sets having the least upper bound property. Equivalence of compactness, limit point compactness and sequential compactness in metrizable spaces. **(12 Hours)**

Unit II - Countability and separation axioms:
The countability axioms and their properties, study of countability properties of spaces $\mathbb{R}_\ell$, $\mathbb{R}_\ell^2$, I_0^2, S_Ω and $\overline{S_\Omega}$. The separation axioms and their properties, separation properties of spaces $\mathbb{R}_K$, S_Ω and $S_\Omega \times \overline{S_\Omega}$. Urysohn lemma(Statement only), Imbedding theorem and Urysohn Metrization theorem, Partitions of unity (finite case), Imbeddings of manifolds. **(12 Hours)**

Unit III - Metrization theorems and paracompactness:
Local finiteness. The Nagata-Smirnov Metrization Theorem. Paracompactness. **(12 Hours)**

Unit IV - The fundamental group and covering spaces:
Homotopy of paths, The fundamental group, Covering spaces, The fundamental group of the circle. **(12 Hours)**

References

[1] J. R. Munkres, *Topology*, 2nd Ed., Pearson Education, Inc, 2000.

[2] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[3] S. Willard, *General Topology*, Addison Wesley, New York, 1968.

[4] J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.

[5] J. L. Kelley – General Topology, Van Nostrand Reinhold Co., New York, 1955.

[6] E. H. Spanier, *Algebraic Topology*, McGraw-Hill, 1966.

MTS 556	Advanced Discrete Mathematics	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 401 (Algebra – I).

Unit I
Basic Counting Principles: Number of one-one functions, Permutations, Combinations, Number of onto functions. Partitions and Stirling Numbers of Second kind.
Advanced Counting: Pigeon-hole Principle, Inclusion-Exclusion Principle, Putting Balls

into boxes, Round Table Configurations, Counting using Lattice Paths, Catalan Numbers. Recurrence Relations, Generating Functions, Using generating functions to prove results related to certain binomial coefficients. **(18 Hours)**

Unit II - Applications of Group Theory:
Recapitulation of Group Action, Orbit Stabilizer Theorem and its applications to Polya's Counting Principle (Polya's Theorem (Special Case) and Polya's Theorem (General Case)) and Polya's Inventory Problems. **(10 Hours)**

Unit III - Boolean Algebras and Switching Functions:
Introduction, Boolean Algebras, Boolean Functions, Switching Mechanisms, Minimization of Boolean Functions, Applications to Digital Computer Design. Switching Functions: Disjunctive and Conjunctive Normal Forms, Gating Networks, Minimal sums of products, Karnaugh Maps and further Applications. **(10 Hours)**

Unit IV - Graph Theory:
Introduction, Matching and Factorization. Extremal Graph Theory - Turans Theorem. DFS, BFS, Shortest Path Algorithms. **(10 Hours)**

References

[1] D. I. A. Cohen, *Basic Techniques of Combinatorial Theory*, John Wiley and Sons, New York, 1978.

[2] G. E. Martin, *Counting: The Art of Enumerative Combinatorics*, UTM, Springer, 2001.

[3] Ralph P. Grimaldi, *Descrete Combinatorial Mathematics*, 5th Ed., Pearson, 2006.

[4] Mott J. L. , Kandel A. and Baker T. P., *Discrete Mathematics for Computer Scientists and Mathematicians*, 2nd Ed., Prentice Hall India, 1986.

[5] Kenneth H. Rosen, *Discrete Mathematics and its Applications*, 7th Ed., McGraw Hill, 2012.

[6] F. Bukley and Frank Harary, *Distance in Graphs*, Addition Wisley Publishing Comany, 1990.

[7] G. Chartrand, L. Lesniak and P. Zhang, *Graphs and Digraphs*, 5th Ed., CRC Press, 2011.

MTS 557	Algebraic Number Theory	4 Credits (48 hours)

Prerequisite: Knowledge of syllabi prescribed for the courses MTH 452 (Algebra – II).

Unit I - Algebraic Numbers:
Recapitulation of Field Extensions and properties, Definition and Examples of algebraic and transcendental numbers, Liouville's Theorem, Algebraic Number Fields, Conjugates and Discriminants, Algebraic Integers, Integral Bases, Norms and Traces, Rings of Integers, Quadratic Fields and Cyclotomic Fields. **(12 Hours)**

Unit II - Factorization into Irreducibles:
Trivial Factorizations, Factorization into Irreducibles, Examples of Non-Unique Factorization

into Irreducibles, Prime Factorization, Euclidean Domains, Euclidean Quadratic Fields, Consequences of Unique Factorization, The Ramanujan-Nagell Theorem. **(12 Hours)**

Unit III - Factorization of Ideals:
Dedekind domains, Fractional ideals, Invertible ideals, Prime factorization of ideals, Congruences, Norm of an ideal, Ideals in different number fields, Ramification index and degree of a prime ideal, The splitting of rational primes in algebraic number fields, Splitting of primes in quadratic fields. **(16 Hours)**

Unit IV - Class Group and Class Number:
Definition of the Class group and class number, Minkowski's theorem, Finiteness of the class-group, Class number computations. **(8 Hours)**

References

[1] I. N. Stewart and David Tall, *Algebraic Number Theory and Fermat's Last Theorem*, A. K. Peters Ltd., 2002.

[2] Jody Esmonde and M. Ramamurthy, *Problems in Algebraic Number Theory*, 2nd Ed. Springer Verlag, 2004.

[3] Pierre Samuel, *Algebraic Theory of Numbers*, Dover Publications, 2008.

[4] Karlheinz Spindler, *Abstract Algebra with Applications*, Vol. II, Rings and Fields, Marcel Dekkar, Inc, 1994.

[5] Saban Alaca and Kenneth S. Williams, *Introductory Algebraic Number Theory*, Cambridge University Press, 2004.

MTS 558	Calculus of Variations and Integral Equations	4 Credits (48 hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Unit I - Variational Problems with the Fixed Boundaries:
Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional. Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples. **(12 Hours)**

Unit II - Variational Problems with Moving Boundaries, Sufficiency Conditions:
Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction of extremals, examples,

One-sided variations, conditions for one sided variations. Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving several variables and their derivatives, Isoperimetric problems, examples. **(12 Hours)**

Unit III - Integral Equations:
Introduction, Definitions and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation.
Fredholm's Integral equations - Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations. **(10 Hours)**

Unit IV
Voltera Integral Equations: A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel.
Fredholm's theory - Hilbert-Schmidt Theorem: Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigenfunction expansion, Hilbert-Schmidt theorem.
Fredholm and Volterra Integro-Differential Equation: Fredholm and Volterra Integro-Differential equation, Singular and nonlinear Integral Equation. **(14 Hours)**

References

[1] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol I, Interscience Press, 1953.

[2] L. E. Elsgolc, *Calculus of Variations*, Pergamon Press Ltd., 1962.

[3] R. Weinstock, *Calculus of Variations with Applications to Physics and Engineering*, Dover, 1974.

[4] D. Porter and D. S. G. Stirling, *Integral Equations, A practical treatment from spectral theory and applications*, Cambridge University Press, 1990.

[5] R. P. Kanwal, *Linear Integral Equations Theory and Practise*, Academic Press 1971.

[6] A. M. Wazwaz, *A first course in integral equations*, World Scientific Press, 1997.

[7] C. Cordumeanu, *Integral Equations and Applications*, Cambridge University Press, 1991.

MTS 559	Mathematical Statistics	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate level.

Unit I - Probability, Conditional Probability and Moments:
Sample space, class of events; Classical and Axiomatic definitions of Probability, their consequences. Conditional Probability, Independence, Bayes' theorem and applications. Random Variables, Distributions Functions, Probability Mass functions, Probability Density functions. Expectations, Moment generating function, Probability generating function,

Chebyshev's and Jensen's inequalities and applications. **(12 Hours)**

Unit II - Distributions:
Standard discrete distribution and their properties – Bernoulli, Binomial, Geometric, Negative Binomial, Poisson distributions. Standard continuous distribution and their properties – Uniform, Exponential, Normal, Beta, Gamma distributions. Functions of random variables – transformation technique and applications, Sampling distributions - t, Chi-square, F and their properties. **(14 Hours)**

Unit III - Random Sequences, Statistical Inference and Testing Hypothesis:
Sequences of random variables - Convergence in distribution and in probability, Chebyshev's, Weak law of large numbers. Central limit theorem and applications. Point estimation-sufficiency, unbiasedness, method of moments, maximum likelihood estimation. Testing of hypotheses – Basic concepts, Neyman-Person lemma, MP test. Likelihood ratio tests, t- test, Chi-square test and their applications. Nonparametric tests and their applications – Sign, Wilcoxon sign rank test, Run test. **(22 Hours)**

References

[1] Rohatgi V. K., *An introduction to probability theory and mathematical statistics*, Wiley Eastern ltd, 1985.

[2] Bhat B. R., *Modern Probability Theory, an introductory text*, Wiley eastern Ltd, 1981.

[3] Robert B Ash, *Probability and Mathematical Statistics*, Academic Press, Inc. NY, 1972.

[4] Hogg R.V. and Craig A. T., *Introduction to Mathematical Statistics*, 6th Ed., McMillan and Co., 2004.

[5] E. L. Lehmann and J. P. Romano, *Testing Statistical Hypothesis*, 3rd Ed., Springer, 2005.

[6] Freund, J.F., *Mathematical Statistics*, 8th Ed., Prentice Hall India, 2012.

Semester wise distribution of credits for M.Sc. Mathematics Programme

SEM	Theory(HC†)		Theory (SC‡)		Open Elective		Lab Credits (SC‡)	Project Credits (HC†)	Total Credits
	No. of Courses	Credits	No. of Courses	Credits	No. of Courses	Credits			
I	3	4	2	4	-	-	2	-	22
II	3	4	2	4	1	3	2	-	22+3*
III	3	4	2	4	1	3	2	-	22+3*
IV	2	4	2	4	-	-	-	4	20
Total		44		32	-	6	6	4	86+6*

†HC - Hard core, ‡SC - Soft core, *Not included for CGPA

Total Hard Core Credits is 44+4=48 (55.81%) and total Soft Core Credits is 32+6=38 (44.19%).

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