Reg. No. | | | | | | | | | |

MTS 507

Third Semester M.Sc. Degree Examination, December 2018/January 2019
MATHEMATICS
Graph Theory
Choice Based Credit System – New Syllabus

Time : 3 Hours   Max. Marks : 70

Note : *1) Answer **any five full** questions.*
*2) Answer to **each full** question shall **not** exceed **eight** pages of the answer book. No additional sheets will be provided for answering.*
*3) Use of scientific calculator is **permitted**.*

1. a) Prove that the maximum number of lines among all p point graphs with no triangles is $[p^2/4]$.

   b) If G is connected graph with p > 3 points, then show that the intersection number w(G) = q if and only if G has no triangles.

   c) Suppose G_1 and G_2 are (p_1, q_1) and (p_2, q_2) graphs respectively. Then show that the product graph $G_1 \times G_2$ has $p_1q_2 + p_2q_1$, lines. **(4+6+4)**

2. a) Let G be a connected graph with $p \geq 3$. Then show that the following statements are equivalent :

   1) G is a block.
   2) Every two points of G lie on a common cycle.
   3) Every point and line of G lie on a common cycle.
   4) Every two lines of G lie on a common cycle.
   5) Given two points and one line of G, there is a path joining the points which contains the line.
   6) For every three distinct points of G, there is a path joining any two of them which contains the third.
   7) For every three distinct points of G, there is a path joining any two of them which does not contain the third.

P.T.O.

b) Show that a cubic graph has a cut point if and only if it has a bridge.

c) Let G be a connected graph of order $P \geq 3$ points without bridges. Suppose that for every line e of G each line of G – e is a bridge. What is G ? Justify your answer. **(8+3+3)**

3. a) Define a tree. Draw all possible trees of order 6. Further show that a graph G is a tree if and only if every two points of G are connected by a unique path.

b) Let G be a graph of order p and size q. If G satisfies any two of the properties

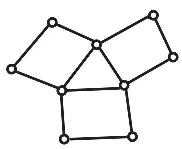
1) G is connected.

2) G is acyclic

3) $q = p - 1$

then show that G is tree.

c) Show that every connected graph has a spanning tree. Further draw all possible spanning trees of G below : **(5+5+4)**

G : 

4. a) If k(G), λ(G) and δ(G) represents point connectivity line connectivity and minimum degree of G respectively. Then show that for every graph G,

$k(G) \leq \lambda(G) \leq \delta(G)$.

b) If G is a cubic graph then show that $k(G) = \lambda(G)$. Further draw a cubic graph such that $k(G) = \lambda(G) = 1$.

c) Suppose G is a graph order P and size $q \geq p - 1$.

Then show that $k(G) \leq \left[\frac{2m}{n}\right]$. **(6+6+2)**

5. a) Prove that a nontrivial connected graph G is Eulerian if and only if every point of G has even degree.

b) Let G be a graph of order $p \geq 3$. If $\deg v \geq p/2$ for each point v of G, then show that G is Hamiltonian. **(7+7)**

6. a) Show that a graph is planar graph if and only if each of its blocks is planar.
   b) Show that the Petersen's graph is non-planar.
   c) Prove that if G is a planar graph of order $p \geq 3$ and size q then $q \leq 3p - 6$. **(6+4+4)**

7. a) Prove that for every graph G, Chromatic number χ (G) of G is, $\chi(G) \leq 1 + \Delta(G)$

      where $\Delta(G)$ is a maximum degree of G.
   b) What is the chromatic number of a tree ? Further give an example of a planar graph with chromatic number 5.
   c) Show that for every graph G,

      $\chi(G) \leq 1 + \max\{\delta(H)\}$.

      where maximum is taken over all induced subgraphs H of G. **(4+4+6)**

8. a) Show that every planar graph is 5-colorable.
   b) Find the chromatic polynomial f(G, t) of the following graph G :

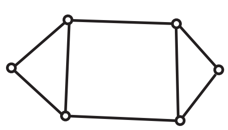
G : 

Fig G : G (6,8) **(7+7)**