MTS 560	Computational Geometry	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate level.

Course Outcome: This course provides an account of fundamental concepts of quantitative geometry and graphical techniques of geometric constructions.

Course Specific Outcome: At the end of the course, the student will be able to

- Understand the adapted frame field
- Derive the basis formulas, the second structural equation
- Understand Intrinsic geometry of surfaces
- Compute first and second structural equations
- Understand different construction methods for conformal geometric surfaces and derive a
- formula for the Gaussian curvature of these conformal geometric surfaces
- Describe and construct basic geometric shapes and concepts by computational means
- Understand and apply Bezier curves in Computer graphics

Unit I - Shape Operators:

The shape operator $M \subseteq R^3$, Normal curvature, Gaussian curvature, Computational Techniques, The implicit case.

(8 Hours)

Unit II - Geometry of Surfaces in $\mathbf{R}^3$

The fundamental equations, Adapted frame field, Form computations, Some global theorems, Leibmann theorem, Isometries and local isometries, Intrinsic geometry of surfaces in $\mathbf{R}^3$, Orthogonal coordinates, Congruence of surfaces.

(10 Hours)

Unit III - Riemannian Geometry

Geometric surfaces, Construction methods, Conformal change, Pull back, Coordinate description, Gaussian curvature, Theorem aegregium, Examples: flat torus, stereographic sphere, the stereographic plane, hyperbolic plane, the projective plane, tangent surfaces.

Covariant derivative: covariant derivative of $\mathbf{R}^2$, parallel vector field, Geodesics, complete geometric surface, Liouville's formula.

(12 Hours)

Unit IV- Computer aided geometric design

Bezier curves, de casterljau algorithm, properties of Bezier curves, Bloossom. Bernstein form of a Bezier curve, Derivative of Bezier curve, Subdivision, Bloossom and polar, Degree elevation, Variation diminishing property, Degree reduction, Non parametric curves, Cross plots, Different forms of a Bazier curve, Weierstrass approximation theorem, Formulas for Berstein polynomials.

(10 Hours)

Unit V- Computer aided geometric design

Interpolation by polynomial curves, Aitken's algorithm, spline curves in Bazier form, Smoothness conditions. C^1 and in C^2 continuity conditions, C^1 quadratic and C^2 cubic B-spline curves, parametrization, C^1 piecewise cubic interpolation, Cubic spline interpolation.

(8 Hours)

References

[1] Barrett O' Neil, *Elementary differential geometry*, Academic Press, New York and London, 2000

[2] G Farin, *Curves and Surfaces for Computer Aided geometric Design*, Academic Press, 1996.

[3] D. J. Struik, *Lectures on Classical Differential Geometry*, Addison Wesley Reading, Massachusetts, 1961.

[4] L. P. Eisenhart, *Riemannian Geometry*, Princeton University Press, Princetion, New Jersey, 1949.

[5] R. L. Bishop and S. J. Goldberg, *Tensor analysis on manifolds*, Macmillan co., 1968