Reg. No. | | | | | | | | | |

BSCMTC 153

Credit Based II Semester B.Sc. Degree Examination, September 2022
(2018 – 19 & Earlier Batches)
MATHEMATICS
Calculus, Group Theory and Differential Equations

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer* ***any ten*** *questions from Part* ***A****.* ***Each*** *question carries* ***3*** *marks.*

2) Answers to Part ***A*** *should be written in the* ***first few*** *pages of the answer book before answers to Part* ***B****.*

3) Answer ***five full*** *questions from Part* ***B*** *choosing* ***one full*** *question from* ***each*** *Unit.*

4) Scientific calculators are ***allowed****.*

PART – A

Answer **any ten** questions : **(10×3=30)**

1. Find a value of c, satisfying mean value theorem for the function, $f(x) = x^2 + 2x - 1$ in [0, 1].

2. Find $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$.

3. Convert $(x - 2)^2 + y^2 = 4$ to polar form.

4. Find the volume of the solid generated by revolving the region between Y axis and the curve $x = 2\sqrt{y}$ about Y axis where $0 \leq y \leq 4$.

5. Find the volume of the solid generated by revolving the region bounded by y = x , y = 1 and x = 0 about the x-axis by Washer method.

6. Find the length of the curve $y = x^{3/2}$, $0 \leq x \leq 1$.

7. If G is a group and $a \in G$, $b \in G$ then prove that
   i) $(a^{-1})^{-1} = a$ ii) $(ab)^{-1} = b^{-1}a^{-1}$.

8. If H and K are subgroups of a group G prove that $H \cap K$ is also a subroup of G.

P.T.O.

9. Write the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 1 & 5 & 7 & 2 & 4 & 3 \end{pmatrix}$$

as a product of transpositions.

10. Solve $\frac{dy}{dx} = xy^2$.

11. Check the exactness of the differential equation
$(2xy - 3x^2)dx + (x^2 + y)dy = 0$.

12. Find the integrating factor of $(y - \cos^2 x)dx + \cos x\, dy = 0$.

13. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = c$.

14. Solve $x^2p^2 - y^2 = 0$.

15. Solve $y = px + p^3$.

PART – B

Unit – I

1. a) State and prove Cauchy mean value theorem. **6**

   b) Evaluate :

   i) $\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

   ii) $\lim_{x \to 0} \frac{x(\cos x - 1)}{\sin x - x}$. **6**

   c) Find the area of the region in polar co-ordinates shared by polar curve $r = 2(1 - \cos\theta)$ and the circle $r = 2$. **6**

2. a) State and prove Rolle's theorem. **6**

   b) Draw the graph of $r = 1 - \sin\theta$. **6**

   c) Find the length of the cardioid $r = a(1 + \cos\theta)$. **6**

Unit – II

3. a) Find the volume of the solid generated by revolving the region bounded by the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$ by Disk method. **6**

   b) Find the volume of the solid generated by revolving the triangular region formed by the vertices (1, 1) (1, 2) (2, 2) about Y axis by Shell method. **6**

   c) If a curve C is defined parametrically by $x = f(t)$, $y = g(t)$, $a \leq t \leq b$ where f' and g' are continuous but not simultaneously zero on [a, b] and c is traversed exactly once as t increases from a to b then derive the formula for the length of C in the form $L = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\,.\,dt$. **6**

4. a) The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about x-axis to generate a solid. Find the volume of the solid generated by Washer's method. **6**

   b) Find the volume of the solid generated by revolving the region bounded by $x = y - y^3$ and the y-axis about the x-axis using Shell method. **6**

   c) Find the length of the astroid, $y = \sin^3 t$, $x = \cos^3 t$, $0 \leq t \leq 2\pi$. **6**

Unit – III

5. a) Prove that a non empty subset H of a group G is a subgroup iff whenever $a \in H$, $b \in H$, $ab^{-1} \in H$. **6**

   b) Let H and K be subgroups of a group G. Prove that HK is a subgroup of G if and only if HK = KH. **6**

   c) Prove that subgroup of a cyclic group is cyclic. **6**

6. a) Let H and K be finite subgroups of G such that HK is also a subgroup. Then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$. **6**

   b) Prove that an infinite cyclic group has exactly 2 generators. **6**

   c) Express the following permutation as a product of disjoint cycles and state whether it is odd or even.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 7 & 6 & 8 & 1 \end{pmatrix}$$ **6**

Unit – IV

7. a) Solve $xydx + (x^2 + y^2)dy = 0$. **6**

   b) Solve $(1 + y^2)dx + (x^2y + y)dy = 0$. **6**

   c) Solve $(x + 2y - 1)dx - (2x + y - 5)dy = 0$. **6**

8. a) Solve $y(x^3 - y)dx - x(x^3 + y)dy = 0$. **6**

   b) Solve $dx - (1 + 2x \tan y)dy = 0$. **6**

   c) Solve $(2x + 3y - 1)\, dx + (2x + 3y + 2)dy = 0$. **6**

Unit – V

9. a) Find the orthogonal trajectories of the family of $r = a \cos 2\theta$. **6**

   b) Solve $xp^2 - 3yp + 9x^2 = 0$; for $x > 0$. **6**

   c) Solve $y'' = x(y')^3$. **6**

10. a) Find the general and singular solution of $2xp^3 - 6yp^2 + x^4 = 0$. **6**

    b) Solve the equation $yy'' + (y')^2 + 1 = 0$. **6**

    c) A thermometer reading 75°F is taken out where the temperature is 20°F. The reading is 30°F, four minutes later. Find the thermometer reading seven minutes after the thermometer was brought outside. **6**
