Reg. No. | | | | | | | | | |

BSCMTC 253

Credit Based IV Semester B.Sc. Degree Examination, September 2022
(2019 – 20 and Earlier Batches)
MATHEMATICS
Multiple Integrals, Infinite Sequences and Series and Complex Analysis

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer* ***any ten*** *questions from Part –* ***A****.* ***Each*** *question carries* ***3*** *marks.*
2) Answer to Part – ***A*** *should be written in the* ***first*** *few pages of the answer book before answers to Part –* ***B****.*
3) Answer ***five full*** *questions from Part –* ***B*** *choosing* ***one full*** *question from* ***each*** *Unit.*
4) Scientific calculators are ***allowed****.*

PART – A

Answer **any ten** questions : **(10×3=30)**

1. a) Evaluate $\int_0^{\pi}\int_0^{x} x\sin y\, dy\, dx$. **3**

b) Write an equivalent double integral with order reversed for $\int_0^{1}\int_y^{\sqrt{y}} dx\, dy$. **3**

c) Find the average value of $f(x, y) = x\cos xy$ over the rectangle $R : 0 \leq x \leq \pi$, $0 \leq y \leq 1$. **3**

d) Find the singular points of $f(z) = \dfrac{z^3+7}{(z^3-2z+2)(z-3)}$. **3**

e) Show that $\sin(\overline{iz}) = \overline{\sin(iz)}$ if and only if $z = in\pi, n \in I$. **3**

f) Find the domain and range of $f(z) = \dfrac{iz}{z-\overline{z}}$. **3**

g) Prove that $f(z) = e^z$ is an entire function. **3**

h) Prove that $|\sinh z|^2 = \sinh^2 x + \sin^2 y$. **3**

i) Find the zeros of $\sin(iz)$. **3**

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j) Find the sum of the series $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n$. **3**

k) Find $\lim_{n\to\infty}\frac{5^n}{7^n}$. **3**

l) Find the Taylor polynomial of order two generated by $f(x)=\frac{1}{x}$ at a = 2. **3**

m) If $\sum_{n=1}^{\infty}|a_n|$ converges, then prove that $\sum_{n=1}^{\infty}a_n$ converges. **3**

n) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+3)!}{3!n!3^n}$. **3**

o) State the limit comparison test. **3**

PART – B

Unit – I **(6×3=18)**

2. a) Evaluate : **6**

i) $\int_0^1\int_0^{\pi}\int_0^{\pi} y\sin z\,dx\,dy\,dz$ ii) $\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dz\,dy\,dx$.

b) Find the area bounded by the Cardioid r = a(1 + cosθ). **6**

c) Change the Cartesian integral $\int_0^6\int_0^y x\,dx\,dy$ into an equivalent polar integral and then evaluate it. **6**

3. a) Evaluate $\iint_R f(u,v)\,dv\,du$ where $f(u,v)=v-\sqrt{u}$ over the triangular region cut from the first quadrant in the uv plane by the line u + v = 1. **6**

b) Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below the region enclosed by the parabola $y = 2 - x^2$ and the line y = x in the xy-plane. **6**

c) Integrate $f(x,y)=\frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}$ over the region $1 \le x^2 + y^2 \le e$, by converting it into a polar integral. **6**

Unit – II **(6×3=18)**

4. a) Find all the roots of $(-8 - i\,8\sqrt{3})^{1/4}$ and represent them geometrically. **6**

b) Let the function f(z) = u(x, y) + iv(x, y) be defined in some neighbourhood of z = a + ib. If the first order partial derivatives of u and v are continuous at (a, b) and if they satisfy Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$, then prove that $f'(z)$ exists at a + ib. **6**

c) Show that $f(x + iy) = e^y \cos x + 2ie^y \sin x$ is not analytic anywhere. **6**

5. a) Let f(z) = u(x, y) + iv(x, y), $z_0 = x_0 + iy_0$ and $w_0 = p + iq$, then prove that $\lim_{z \to z_0} f(z) = w_0 \Leftrightarrow \lim_{(x, y) \to (x_0, y_0)} u(x, y) = p$ and $\lim_{(x, y) \to (x_0, y_0)} v(x, y) = q$. **6**

b) If z_1 and z_2 are any two complex numbers, then prove that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. **6**

c) Using the definition of limit, show that $\lim_{z \to z_0} (az^2 + bz + c) = az_0^2 + bz_0 + c$. **6**

Unit – III **(6×3=18)**

6. a) Show that the function $u(x, y) = -x^3 + 3xy^2 + 2y + 1$ is harmonic and construct the corresponding analytic function. **6**

b) Show that $|\sinh z|^2 = \sinh^2 x + \sin^2 y$. **6**

c) Find the primitive period and zeros of sinh(iz + 2). **6**

7. a) Find the general values of **6**

i) log i ii) log e.

b) Evaluate $\int_\gamma f(z)dz$ where γ is the arc from z = – 1 – i to z = 1 + i, consisting of a line segment from (– 1, – 1) to (0, 0) and portion of the curve $y = x^3$. **6**

c) Prove that the function f(z) = u + iv is analytic in D if and only if u and v are harmonic conjugates of each other. **6**

Unit – IV **(6×3=18)**

8. a) Using partial fractions, find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$. **6**

b) Test the convergence of the sequence whose n^{th} term is **6**

i) $a_n = \frac{\log n}{n}$ ii) $a_n = \frac{n}{2^n}$.

c) Discuss the convergence of $a_n = \left(\frac{n+1}{n-1}\right)^n$. **6**

9. a) Find the Maclaurin series for the function $f(x) = \frac{1}{1+x}$. **6**

b) Express the numbers 1.414414414... as the ratio of two integers. **6**

c) Find whether the following series converge. If so find their sum. **6**

i) $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} + \frac{1}{3^n}\right)$ ii) $\sum_{n=0}^{\infty} \ln\left(\frac{n}{n+1}\right)$.

Unit – V **(6×3=18)**

10. a) State and prove the integral test for convergence of series. **6**

b) Test the convergence of the series : **6**

i) $\sum_{n=1}^{\infty} n!\, e^{-n}$ ii) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$.

c) Discuss whether the series given below converge absolutely or converge conditionally or diverge. **6**

i) $\sum_{n=1}^{\infty} (-1)^{n+1}(0.1)^n$ ii) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$.

11. a) State and prove ratio test for series. **6**

b) Test the convergence of the infinite series : **6**

i) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ ii) $\sum_{n=1}^{\infty} \frac{3^n+1}{5^n}$.

c) Prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + u_5 + \ldots$ converges if all three of the following are satisfied. **6**

i) The u_n's are all positive.

ii) $u_n \geq u_{n+1}$ for all $n \geq N$, for some positive integer N.

iii) $u_n \to 0$.