Reg. No. | | | | | | | | | |

BSCMTC 358

Credit Based VI Semester B.Sc. Degree Examination, September 2022
(2020-21 and Earlier Batches)
MATHEMATICS
Partial Differential Equations, Fourier Series and Linear Algebra

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer **any ten** questions from Part – **A**. **Each** question carries **3** marks.*
*2) Answers to Part – **A** should be written in the **first few** pages of the answerbook before answers to Part – **B**.*
*3) Answer **five full** questions from Part – **B** choosing **one full** question from **each** Unit.*
*4) Scientific **calculators** are allowed.*

PART – A

Answer **any ten** questions : **(10×3=30)**

1. Verify the integrability condition for $(y^2 + yz)\, dx + (xz + z^2)\, dy + (y^2 - xy)\, dz = 0$.

2. Eliminate ‘a’ and ‘b’ from $\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1$.

3. Find the complete integral of $p^2 + q^2 = 4$.

4. State the Dirichlet’s conditions for Fourier series expansion of a function.

5. If $f(x)=\begin{cases}1, & 0<x<1\\ -1, & 1<x<2\end{cases}$, find the Fourier coefficient a_0.

6. Write the complex form of the Fourier series of a function and the formula for the complex Fourier coefficient.

7. Determine whether the vectors $V_1 = (1, 0, 2)$, $V_2 = (1, 1, 1)$ and $V_3 = (4, 5, 3)$ of R^3 are linearly independent.

8. If V is a vector space over F, then prove that $a.v = 0 \Rightarrow$ either $a = 0$ or $v = 0$ where $a \in F$ and $v \in V$.

P.T.O.

9. In an inner product space V = R, if v is orthogonal to v′ then prove that $||v + v'||^2 = ||v||^2 + ||v'||^2$ where $v, v' \in V$.

10. If A is a nilpotent matrix, then prove that I + A is non-singular.

11. Let $T : V \to V'$ be a linear transformation. If $V_1, V_2, \ldots, V_n$ are elements of V such that $T(V_1), \ldots, T(V_n)$ are linearly independent. Show that $V_1, V_2, \ldots, V_n$ are linearly independent.

12. Let $T : R^3 \to R^3$ be defined by T(x, y, z) = (x, 2y, x + y + z). Find the dimension of T(V).

13. Find the rank of the matrix $\begin{pmatrix} 2 & 3 & -1 \\ -1 & 0 & 4 \\ 4 & 5 & 8 \end{pmatrix}$ using elementary row operations.

14. Find the characteristic roots of $A = \begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix}$.

15. Prove that every non singular matrix is a product of elementary matrices.

PART – B

Unit – I

1. a) Eliminate the arbitrary function ‘ f ’ from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$. **6**

   b) Solve (y + z) p + (z + x)q = x + y by Lagrange’s method. **6**

   c) Solve : p(1 + q) = qz. **6**

2. a) Assuming integrability condition, solve $(yz + z^2)dx - xzdy + xydz = 0$. **6**

   b) Obtain the partial differential equation of all spheres whose centres lie on the z-axis. **6**

   c) Solve $p^2 + q^2 = x + y$. **6**

Unit – II

3. a) Find the Fourier expansion of the periodic function whose definition in one period is $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$. **9**

   b) Find the half range sine and cosine expansions of the function f(x) = x, 0 < x < 2. **9**

4. a) Expand $f(x) = x^2$ in Fourier series over the interval $(-p, p)$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$. **9**

b) Find the complex form of Fourier series of the function whose definition in one period is $f(x) = e^{-x}$, $-1 < x < 1$. **9**

Unit – III

5. a) Let V be a vector space over F and let $\{w_\alpha\}_{\alpha \in I}$ be a collection of subspaces of V, prove that $W = \bigcap_{\alpha \in I} w_\alpha$ is also a subspace of V. **6**

b) Let V be a vector space of dimension n. Then prove that any set of m linearly independent elements $(m \leq n)$ can be completed to a basis of V. **6**

c) Define an inner product space. Prove that any orthonormal set in an inner product space V is linearly independent. **6**

6. a) Prove that the set $\{V_1, V_2, \ldots, V_n\}$ is a minimal generating set for V if and only if it is a basis of V. **6**

b) Prove that V is a direct sum of subspaces V_1 and V_2 if and only if every $v \in V$ can be expressed uniquely as $v = v_1 + v_2$, $v_1 \in V_1$, $v_2 \in V_2$. **6**

c) Let V be an inner product space, prove that $|< v, v' >| \leq \| v\| \, \| v' \|$, for all $v, v' \in V$. **6**

Unit – IV

7. a) Define a linear transformation. Prove that a linear transformation $T : V \to V'$ is a one to one mapping if and only if Ker T = {0}. **6**

b) Test the columns of $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 0 & 4 \\ 3 & 2 & 0 \\ 4 & 1 & 0 \end{bmatrix}$ for linear independence and hence find its column rank. **6**

c) Prove that a linear transformation T is an isomorphism if and only if m(T) is a non singular matrix. **6**

8. a) Let V and V′ be any two vector spaces of dimension m and n respectively over the field F. Then prove that the space L(V, V′) of all linear transformations of V into V′ is isomorphic onto the space M_{mn} (F) of all m×n matrices over the field F. **6**

b) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$ be the matrix of T ∈ L (R^3, R^3) with respect to the standard basis. Find the matrix of T with respect to the basis { (1, 1, 0), (0, 1, 0), (0, 1, 1) }. **6**

c) Let V be a vector space over F and let W be a subspace of V. Let $V_1, V_2, ..., V_n$ be a basis of V such that $V_1, V_2, ..., V_m$ ($m \le n$) is a basis of W. Then prove that $\bar{V}_{m+1}, \bar{V}_{m+2}, ..., \bar{V}_n$ is a basis of $^V/_W$. **6**

Unit – V

9. a) Find the rank of the matrix $A = \begin{bmatrix} 6 & -2 & 18 \\ -4 & 1 & 11 \\ -5 & 2 & 16 \end{bmatrix}$ using elementary row or column operations. **6**

b) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Find A^{-1} by elementary row or column operations. **6**

c) If $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. Find the minimal polynomial of A. **6**

10. a) Show that the system of equations

$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + 3x_2 + x_3 = 0$$
$$3x_1 + 6x_2 + 5x_3 = 0$$

has only a trivial solution. **6**

b) Define minimal polynomial of a matrix $A \in M_n$ (F). Let $A \in M_n$ (F) and let q(x) ∈ F [x], be the minimal polynomial of A. If f(x) ∈ F[x] is any other polynomial satisfied by A, then prove that q(x) divides f(x). **6**

c) Let $A \in M_n$ (F). Let $\lambda_1, \lambda_2, ..., \lambda_m \in$ F be the distinct characteristic roots of A. $V_1, V_2, ..., V_m$ are the corresponding characteristic vectors, then prove that $V_1, V_2, ..., V_m$ are linearly independent over F. **6**
