Reg. No. | | | | | | | | | |

BSCMTC 359

Credit Based VI Semester B.Sc. Examination, September 2022
(2020 – 21 and Earlier Batches)
MATHEMATICS
Special Paper – VIII (a) : Graph Theory

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer* ***any ten*** *questions from Part –* ***A****.* ***Each*** *question carries* ***3*** *marks.*

2) Answers to Part – ***A*** *should be written in the first few pages of the answer book before answers to Part –* ***B****.*

3) Answer ***five full*** *questions from Part –* ***B*** *choosing* ***one*** *full question from* ***each*** *Unit.*

4) Scientific calculators are ***allowed****.*

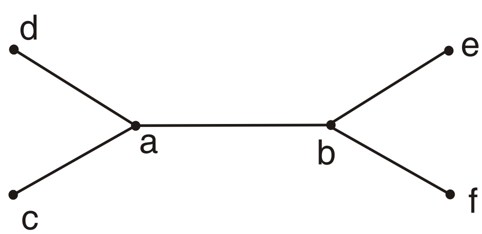
PART – A

1. Define (i) Euler graph (ii) Euler line. **3**

2. If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. **3**

3. Prove that in any tree with two or more vertices there are atleast two pendent vertices. **3**

4. Find the center of the graph by finding eccentricity of each vertex in the following graph. **3**



5. Define spanning tree with an example. **3**

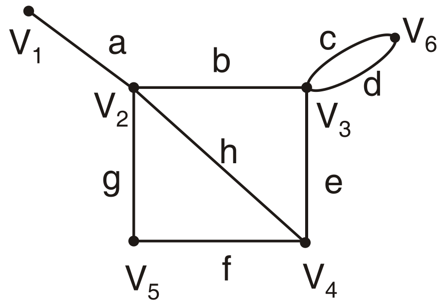
P.T.O.

6. Define (i) Edge connectivity (ii) Vertex connectivity. **3**

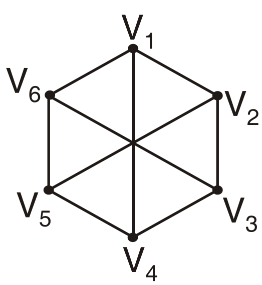
7. Prove that every cutset in a connected graph G must contain atleast one branch of every spanning tree of G. **3**

8. Define separable graph with an example. **3**

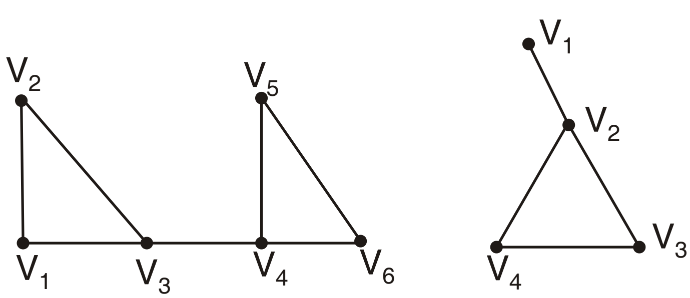
9. Write the path matrix $P(V_1, V_4)$ for the vertices V_1 and V_4 for the given graph. **3**



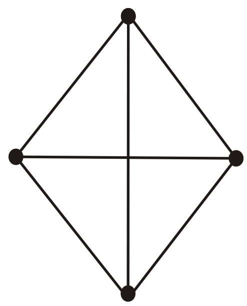
10. Check whether the given graph is a bipartite graph. **3**

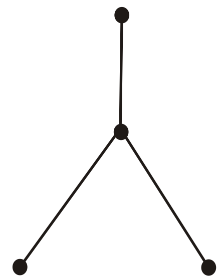


11. Write the chromatic number for the following graphs. **3**



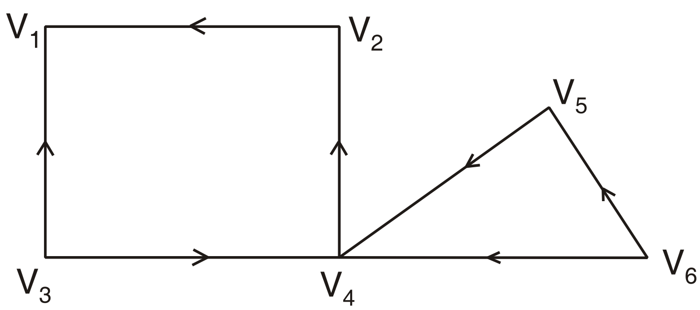
12. Write the chromatic polynomial of the following graphs. **3**





13. Prove that the number of simple labelled graphs with n vertices is $2^{\frac{n(n-1)}{2}}$. **3**

14. Write the indegree and outdegree of each vertex in the following graph. **3**



15. Define (i) Simple digraph (ii) Directed path (iii) Asymmetric digraph. **3**

PART – B

Unit – I

16. a) Prove that a connected graph G is a Euler graph iff G can be decomposed into circuits. **6**

b) Prove that a graph G is disconnected if and only if their vertex set V can be partitioned into two disjoint subset V_1 and V_2 such that there is no edge in G whose one end vertex in V_1 and another vertex in V_2. **6**

c) Define degree of a vertex and regular graph. Prove that the number of vertices of odd degree in a graph is always even. **6**

17. a) Define distance in a graph. Prove that distance between two vertices in a connected graph is a metric. **6**

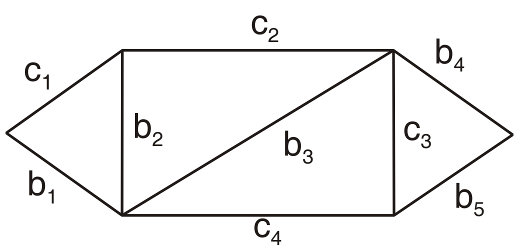
b) Prove that every tree has either one or two centres. **6**

c) Define binary tree. Prove that the number of pendent vertices in a binary tree with n vertices is $\left(\frac{n+1}{2}\right)$. **6**

Unit – II

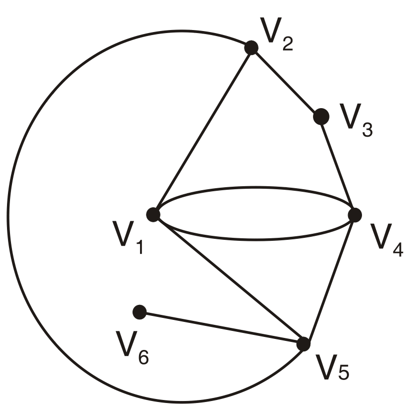
18. a) Prove that every circuit has an even number of edges in common with any cutset. **6**

b) Define a fundamental circuit and list 5 fundamentals circuits of the following graph with respect to a spanning tree $\{C_1, C_2, C_3, C_4\}$. **6**



c) Prove that with respect to a given spanning tree T, a branch b_i that determines a fundamental cutset S is contained in every fundamental circuit associated with the chords in S and in no other. **6**

19. a) Draw a geometric dual of the following graph. **6**



b) Prove that Kuratowski's second graph $K_{3, 3}$ is non-planar. **6**

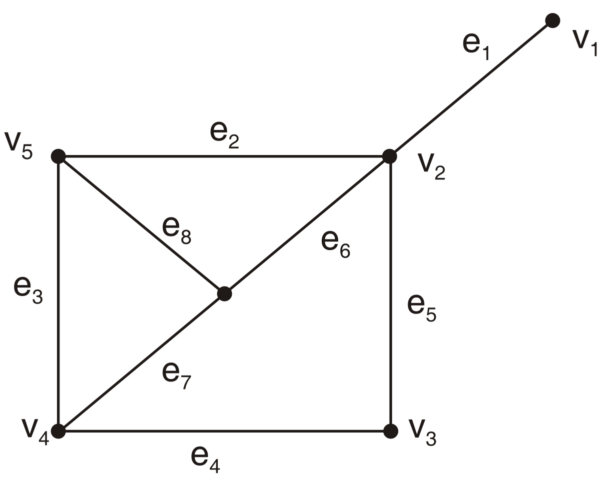
c) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane. **6**

Unit – III

20. a) Prove that the rank of the incidence matrix of a connected graph of n vertices is n – 1. **6**

b) Prove that the ring sum of two circuits in a graph is either a circuit or an edge disjoint union of circuits. **6**

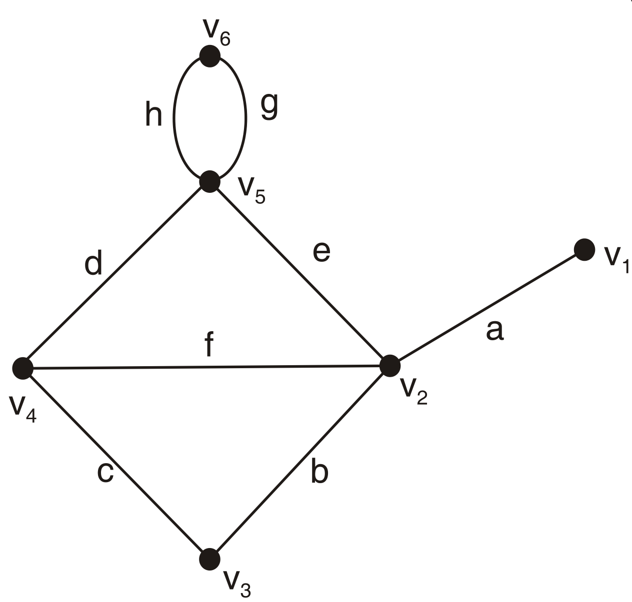
c) Write the adjacency matrix of the following graph. **6**



21. a) Prove that the rank of the cutset matrix C(G) is equal to the rank of the incidence matrix A(G) is equal to the rank of the graph G. **6**

b) Let B and A be respectively the circuit matrix and the incidence matrix of a self loop free graph whose columns are arranged in the same order of edges. Then prove that $A.B^T \equiv 0 \pmod 2$. **6**

c) Write the incidence matrix of the following graph. **6**



Unit – IV

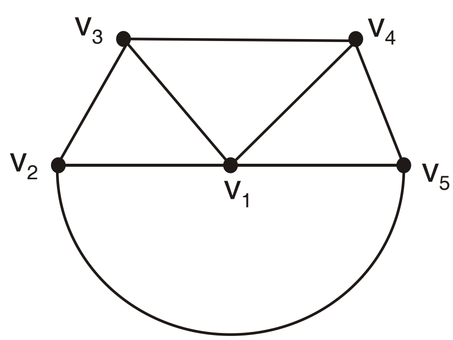
22. a) Prove that every tree with two or more vertices is 2-chromatic. **6**

b) Prove that a graph with atleast one edge is 2 chromatic if and only if it has no circuits of odd length. **6**

c) Prove that the chromatic polynomial of an n vertex tree is

$P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$. **6**

23. a) Find the chromatic polynomial of the following graph : **6**



b) Prove that a graph on n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1) \ldots \lambda -(n-1)$. **6**

c) Define : **6**

i) Proper coloring of a graph

ii) Chromatic number of a graph

iii) Bipartite graph and give an example each.

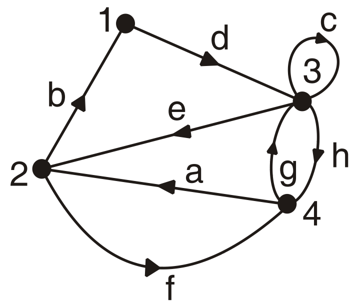
Unit – V

24. a) Prove that the determinant of every square submatrix of the incidence matrix of a digraph is –1, 1 or 0. **6**

b) Let A_f be the reduced incidence matrix of a connected digraph. Prove that the number of spanning trees in a graph equals the value of the det $(A_f . A_f^T)$. **6**

c) Prove that there are n^{n-2} labelled trees with n vertices $n \geq 2$. **6**

25. a) Define a circuit matrix of a digraph. Write circuit matrix of the following digraph. **6**



b) Define : **6**

i) Pendent vertex

ii) Isolated vertex

iii) Symmetric graph

iv) Balanced graph

v) Self loop

vi) Strongly connected graph.

c) Let B and A be respectively the circuit matrix and the incidence matrix of a self-loop free digraph such that columns in B and A are arranged in the same order of edges. Then prove that $A.B^T = B.A^T = 0$. **6**