Reg. No.

BSCMTC 360

Credit Based VI Semester B.Sc. Degree Examination, September 2022
MATHEMATICS (Special Paper – 8b)
Linear Programming and its Applications
(2020 – 21 and Earlier Batches)

Time : 3 Hours Max. Marks : 120

Instructions : *1) Answer* ***any ten*** *questions from (Part –* ***A****).* ***Each*** *question carries* ***3*** *marks.*

2) Answer ***five full*** *questions from (Part –* ***B****) choosing* ***one full*** *question from* ***each*** *Unit.*

3) ***Scientific*** *calculators are* ***allowed****.*

PART – A

1. Define : **(10×3=30)**

   i) A convex set in R^n.

   ii) Closed ball in R^n.

2. Convert the LPP below to the canonical form

   Minimize $g(x, y, z) = x - 2y - z$

   Subject to $10x + 5y + 2z \leq 1000$

   $2x + 7z \leq 800$

   $x, y, z \geq 0$.

3. Pivot on $a_{21} = 4$ in the following canonical maximization tableau.

$$\begin{array}{|cc|c|l}
x_1 & x_2 & -1 & \\ \hline
1 & 3 & 2 & = -t_1 \\
4 & 6 & 7 & = -t_2 \\ \hline
8 & 5 & 6 & = f \\ \hline
\end{array}$$

4. Write the negative transpose of the minimum tableau.

$$\begin{array}{c|ccc|c|}
\cline{2-5}
x_1 & 2 & 3 & 4 & 100 \\
x_2 & 2 & 1 & 7 & 124 \\
x_3 & 1 & 4 & 5 & 228 \\
\cline{2-5}
-1 & 12 & 14 & 25 & 0 \\
\cline{2-5}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{= t_1} & = t_2 & \multicolumn{1}{c}{= t_3} & \multicolumn{1}{c}{= g}
\end{array}$$

5. Given the LPP below, state the dual canonical minimization LPP.

   Maximize $f(x, y) = 5x + 3y$

   Subject to $x + 2y \leq 10$

   $2x + y \leq 15$

   $x, y \geq 0.$

6. Write the matrix reformulation of the canonical maximization LPP.

7. Define complimentary slackness of dual canonical LPP.

8. Reduce the table of the matrix game using domination when $x \leq y$.

$$\begin{bmatrix} 0 & \frac{y}{4} \\ \frac{(x-y)}{4} & 0 \end{bmatrix}$$

9. State Von-Neumann minimax theorem.

10. State the process of converting an un balanced transportation problem when supply is less than the demand.

11. Define a cycle in a table of transportation.

12. Find all permutation set of zeros in the following table of balanced assignment problem.

$$\begin{array}{|ccc|} \hline 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline \end{array}$$

13. Define source, sink and intermediate vertex in a capacitated directed network.

14. Prove that any flow in a capacitated directed network satisfies $\Sigma_j \phi (V_j) = 0$.

15. Define an α – path in a capacitated directed network.

PART – B

Unit – I

1. a) Solve the following LPP graphically. **9**

Minimize $C(x, y) = 300x + 500y$

Subject to $20x + 40y \geq 1000$

$25x + 20y \geq 800$

$x, y \geq 0.$

b) Apply simplex algorithm for the following tableau. **9**

$$\begin{array}{|cc|c|l} \multicolumn{1}{c}{x_1} & \multicolumn{1}{c}{x_2} & \multicolumn{1}{c}{-1} & \\ \hline 1 & 2 & 20 & = -t_1 \\ 2 & 2 & 30 & = -t_2 \\ 2 & 1 & 25 & = -t_3 \\ \hline 200 & 150 & 0 & = f \\ \hline \end{array}$$

2. a) State the complete simplex algorithm for the maximum tableau. **9**

b) Solve using simplex algorithm. **9**

$$\begin{array}{|cc|c|l} \multicolumn{1}{c}{x_1} & \multicolumn{1}{c}{x_2} & \multicolumn{1}{c}{-1} & \\ \hline 2 & 1 & 8 & = -t_1 \\ 1 & 2 & 10 & = -t_2 \\ \hline 30 & 50 & 0 & = f \\ \hline \end{array}$$

Unit – II

3\. a) Solve the following minimization LPP using simplex algorithm. **9**

$$\begin{array}{c|cc|c|} \cline{2-4} x & -2 & 1 & -3 \\ y & 1 & -2 & -2 \\ \cline{2-4} -1 & 1 & 0 & 0 \\ \cline{2-4} \multicolumn{1}{c}{} & = t_1 & = t_2 & \multicolumn{1}{c}{= g} \end{array}$$

b) State the dual Simplex algorithm for the minimum tableau. **9**

4\. a) For any pair of feasible solutions of dual canonical LPP, prove that $g - f = SX' + Y'T$. **9**

b) Solve the following non canonical LPP. **9**

$$\begin{array}{|ccc|c|l} \multicolumn{1}{c}{x} & \textcircled{v} & \multicolumn{1}{c}{z} & \multicolumn{1}{c}{-1} & \\ \hline -1 & 1 & 1 & 6 & = -0 \\ 1 & 1 & 0 & 1 & = -t_1 \\ \hline 1 & 2 & 1 & 0 & = f \\ \hline \end{array}$$

Unit – III

5\. a) Find the optimal strategies for the row and the column player of the matrix game with the pay off matrix $\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$. **9**

b) Solve the following dual non canonical LPP. **9**

$$\begin{array}{c|cc|c|l} \multicolumn{1}{c}{} & \textcircled{x_1} & \multicolumn{1}{c}{x_2} & \multicolumn{1}{c}{-1} & \\ \cline{2-4} \textcircled{y_1} & 2 & -1 & -1 & = -0 \\ y_2 & -1 & 1 & -1 & = -t_1 \\ \cline{2-4} -1 & 2 & 1 & 0 & = f \\ \cline{2-4} \multicolumn{1}{c}{} & = 0 & = s_1 & \multicolumn{1}{c}{= g} & \end{array}$$

6. a) Solve the following dual non canonical LPP. **9**

$$
\begin{array}{c|ccc|c|l}
 & \enclose{circle}{x_1} & \enclose{circle}{x_2} & x_3 & -1 & \\
\hline
\enclose{circle}{y_1} & 1 & -1 & 2 & 1 & = -0 \\
y_2 & 2 & 0 & 2 & -1 & = -t_1 \\
y_3 & 0 & 1 & -1 & -1 & = -t_2 \\
\hline
-1 & 1 & -1 & 3 & 0 & = f \\
\hline
 & = 0 & = 0 & = s_1 & = g &
\end{array}
$$

b) Find the Von-Neumann value of the matrix game. **9**

$$
\begin{bmatrix}
-1 & 1 & -1 & 2 \\
-1 & -1 & 1 & 1 \\
0 & 1 & 1 & -1
\end{bmatrix}
$$

Unit – IV

7. a) State the transportation algorithm to solve a balanced transportation problem. **9**

b) Solve the following assignment problem. **9**

8	7	10
7	7	8
8	5	7

8. a) State the Hungerian algorithm to solve a balanced assignment problem. **9**

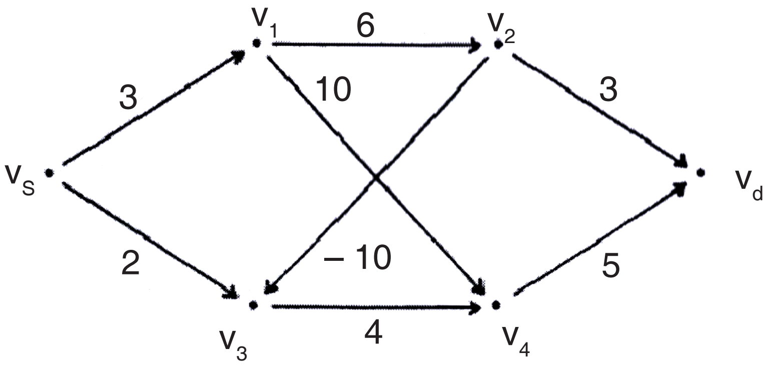
b) Solve the balanced transportation problem below. **9**

7	2	4	10
10	5	9	20
7	3	5	30
20	10	30	

Unit – V

9. a) State the maximal flow algorithm. **9**

b) Solve the shortest path network problem below. Give the shortest path and its value. **9**



10. a) State the shortest path algorithm 1. **9**

b) Solve the maximal flow network problem and the corresponding minimal cut and cut capacity. **9**