Reg. No. | | | | | | | | | |

BSCMTC 382

Choice Based Credit System VI Semester B.Sc. Degree Examination, September 2022
(CBCS) (2021-22 Batch Onwards)
MATHEMATICS
Linear Algebra – VIII (a)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any ten** questions from Part – **A**. **Each** question carries **2** marks.*
*2) Answers to Part – **A** should be written in the **first few** pages of the answer book **before** answers to Part – **B**.*
*3) Answer **any twelve** questions from Part – **B**. **Each** question carries **5** marks.*
*4) Scientific calculators are **allowed**.*

PART – A

I. Answer **any 10** questions : **(10×2=20)**

1) If $T : V \to V'$ is a linear transformation, then prove that Ker T is a subspace of V.

2) Let $T : V \to V$ be a linear transformation of V onto V, then prove that T is (1, 1) mapping if and only if T is onto.

3) Let $V = R^2 = V'$ and let $T : V \to V'$ be a linear transformation which is rotated through θ° about the origin with respect to the basis {(1, 0), (0, 1)} of V and V', find the matrix of linear transformation.

4) Show that the ring $M_n(F)$ has zero divisors.

5) Find the row rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 3 \\ -3 & -6 & -9 & -3 \end{bmatrix}$.

6) Define the rank of a linear transformation.

P.T.O.

7) If A and B are similar matrices, then prove that rank A = rank B.

8) If A, B $\in M_n(F)$, then show that , rank (A + B) = rank A + rank B by giving an example.

9) If $\sum_{j=1}^{n} a_{ij} x_j = 0$, i = 1, 2, ..., m be a system of 'm' homogeneous linear equations in 'n' unknowns. If m < n, prove that the system always has a nontrivial solution.

10) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

11) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

12) Show that the characteristic roots of $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ are the fourth roots of unity.

13) If A $\in M_n(F)$ and A^T be the transpose of A, then prove that $P_A(t) = P_{A^T}(t)$.

14) If A $\in M_n(F)$ is a nilpotent matrix, then prove that all its characteristic roots are zeros.

PART – B

II. Answer **any 12** questions : **(12×5=60)**

1) Prove that a linear transformation $T : V \to V'$ is (1, 1) mapping if and only if KerT = {0}.

2) Let $T : V \to V'$ be a linear transformation of V onto V', with KerT = W. Prove that $\frac{V}{W} \cong V'$.

3) Show that a linear transformation $T : V \to V'$ is (1, 1) if and only if it maps linearly independent set in V onto linearly independent set in V'.

4) Let V = C, the space of complex numbers and $V' = R^2$. Prove that the mapping $T : V \to V'$ defined by T(a + ib) = (a, b) is a linear transformation. Also prove that T is an isomorphism.

5) Show that if A is nilpotent matrix then (I + A) is non-singular.

6) Let $V = R^3$ and let $T : V \to V'$ be a linear transformation defined by $T(x, y, z) = (x', y', z')$ where $x' = 2x$, $y' = 4y$, $z' = 5z$. Find the matrix of T with respect to the basis $\left(2/3, 0, 0\right), \left(0, \frac{1}{2}, 0\right), \left(0, 0, \frac{1}{4}\right)$ of V.

7) Let $V = V' = R^3$ and let $T : V \to V'$ be a linear transformation defined by $T(x, y, z) = (x', y', z')$ where $x' = x$, $y' = 2y$, $z' = x + y + z$. Find the rank of T.

8) Using elementary row operations, find the row rank of the matrix A,

$$A = \begin{bmatrix} 6 & -2 & -18 \\ -4 & 1 & 11 \\ -5 & 2 & 16 \end{bmatrix}.$$

9) Find A^{-1} if $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ by performing elementary row operations.

10) Show that the system of equations has only trivial solution

$x_1 + x_2 + x_3 = 0$

$2x_1 + 3x_2 + x_3 = 0$

$3x_1 + 6x_2 + 5x_3 = 0.$

11) Prove that a system of n, non homogeneous equations in n unknowns has a unique solution if and only if the associate matrix is non singular.

12) Find the dimension of the space of solutions if

$2x_1 + x_2 - 3x_3 - x_4 = 0$

$x_1 + 2x_3 = 0$

$3x_1 - x_2 - x_3 + x_4 = 0.$

13) Let $A \in M_n(F)$ and let $q(x) \in F(x)$ be a minimum polynomial of A. If $f(x) \in F[x]$ is any other polynomial satisfied by A, then prove that q(x) divides f(x).

14) Let $A \in M_n(F)$ with $q(x) = a_0 + a_1x + a_2x^2 + ... + x^m$ be the minimum polynomial of A. Then prove that A is non singular if and only if $a_0 \neq 0$.

15) Find the minimum polynomial of the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -9 \\ 0 & 1 & 6 \end{bmatrix}$.

16) Let $A \in M_n(F)$ and $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_m \in F$ be distinct characteristic roots of A. If $v_1, v_2, v_3, ..., v_m$ are the corresponding characteristic vectors, then prove that $v_1, v_2, v_3, ..., v_m$ are linearly independent.

17) Find the characteristic roots of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

18) If A and $B \in M_n(F)$ are such that B is similar to A, then prove that $P_A(t) = P_B(t)$.
