Reg. No. | | | | | | | | | |

BSCMTCN 201

II Semester B.Sc. Degree Examination, September 2022
(NEP 2020) (2021 – 22 Batch Onwards)
MATHEMATICS
Number Theory – II, Algebra – II and Calculus – II (DSCC)

Time : 2 Hours Max. Marks : 60

Instructions : 1) *Answer **any ten** questions from Part – **A**. **Each** question carries **2** marks.*
2) *Answers to Part – **A** should be written in the first few pages of the answer book before answers to Part – **B**.*
3) *Answer **any eight** questions from Part – **B**, choosing **two** questions from **each** Unit. **Each** question carries **5** marks.*
4) *Use of scientific calculator is **permitted**.*

PART – A **(10×2=20)**

1. If p is a prime, then prove that $a^p \equiv a(\text{mod } p)$ for any integer a.

2. If p is a prime and k > 0, then prove that $\phi(p^k) = p^k - p^{k-1}$.

3. Calculate ϕ (1001).

4. If n = 160, find the sum of integers less than n and relatively prime to n.

5. In a group G, prove the following :
   i) $(a^{-1})^{-1} = a, \forall a \in G$.
   ii) $(ab)^{-1} = b^{-1}a^{-1}, \forall a, b \in G$.

6. If H and K are subgroups of G, then prove that $H \cap K$ is also a subgroup of G.

7. Prove that every cyclic group is an abelian group.

8. Find $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

P.T.O.

9. Find $\frac{\partial^2 f}{\partial x^2}$ if $f(x, y) = x^2 + 3xy + y - 1$.

10. Find $\frac{\partial z}{\partial x}$ if the equation $yz - \ln z = x + y$.

11. Evaluate $\int_C (3x^2 - 2y + z)ds$, where C is the line segment joining from (0, 0, 0) to (2, 2, 2).

12. Evaluate $\int_0^{\pi}\int_0^{x} x \sin y \, dy \, dx$.

13. Evaluate $\iint_R dy \, dx$, where R is the region bounded by $y = 2x^2$ and $y^2 = 4x$.

14. Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx \, dz \, dy$.

PART – B

Unit – I

15. State and prove Euler's theorem.

16. If p is a prime, then prove that $(p - 1)! \equiv - 1 (\mathrm{mod}\ p)$.

17. For each positive integer $n \geq 1$, then prove that $n = \sum_{d|n} \phi(d)$, the sum being extended over all positive divisors of n.

18. Express $\frac{187}{57}$ as a finite simple continued fraction.

Unit – II

19. Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G, if and only if, HK = KH.

20. Prove that a subgroup of cyclic group is a cyclic group.

21. Let H and K be subgroups of a group G such that HK is a subgroup of G, then prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

22. Define a centre of a group. Prove that the centre Z(G) of a group G is a subgroup of G.

Unit – III

23. Applying the two-path test, show that the function $f(x,y)=\dfrac{2x^2y}{x^4+y^2}$ has no limit as $(x, y) \to (0, 0)$.

24. If $f(x, y) = x \cos y + ye^x$, find $\dfrac{\partial^2 f}{\partial x^2}, \dfrac{\partial^2 f}{\partial y \partial x}, \dfrac{\partial^2 f}{\partial y^2}, \dfrac{\partial^2 f}{\partial x \partial y}$.

25. If $w = x + 2y + z^2, x = \dfrac{r}{s}, y = r^2 + \ln s, z = 2r$, then find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$.

26. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

Unit – IV

27. Find the volume of the prism whose base is the triangle in the xy – plane bounded by the x – axis and the lines y = x and x = 1 and whose top lies in the plane.

28. Evaluate $\iint_R f(x,y)dA$, where $f(x, y) = x^2 + y^2$ and R is the region at triangle with vertices (0, 0), (1, 0) and (0, 1).

29. By changing Cartesian integral into polar co-ordinates, evaluate $\iint_R e^{x^2+y^2} dy\, dx$, where R is the semicircular region bounded by the x – axis and the curve $y=\sqrt{1-x^2}$.

30. Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\, dy\, dx$.