Reg. No. | | | | | | | | | |

BSCSTC 281

Choice Based Credit System IV Semester B.Sc. Degree Examination, September 2022
(2020-21 Batch Onwards)
STATISTICS
Statistical Inference – I

Time : 3 Hours Max. Marks : 80

Instructions : *1) A single booklet containing* ***40*** *pages will be issued.*
2) ***No*** *additional sheets will be* ***issued****.*

PART – A

Answer **any ten** of the following : **(10×2=20)**

1. a) Show that if T_n is unbiased for θ and $g(\theta)$ is a linear function of θ. Then $g(T_n)$ is unbiased for $g(\theta)$.

   b) State the necessary and sufficient condition for consistency of an estimator.

   c) State Fisher-Neyman criteria for sufficiency.

   d) State any two properties of maximum likelihood estimators.

   e) Obtain $100(1-\alpha)\%$ confidence interval for the mean of a population based on large sample.

   f) Define Type I and Type II errors.

   g) State any two properties of likelihood ratio test procedure.

   h) Briefly explain the procedure of testing ratio of variances of single normal populations with unknown means.

i) When do you say that a test procedure is unbiased ?

j) Briefly explain the procedure of testing ratio of variances of two normal populations with unknown means.

k) What do you mean by size of a critical region ? Also explain the concept of power function.

l) Mention any two applications of t-distribution in testing of hypothesis.

PART – B

Answer **any five** of the following : **(5×6=30)**

2. Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$. Show that $X_{(n)}$ is biased for θ. Find a function of $X_{(n)}$ which is unbiased for θ. Is $X_{(n)}$ asymptotically unbiased ?

3. For the density function $f(x, \theta) = \frac{\theta^{\alpha}}{\Gamma\alpha} e^{-\theta x} x^{\alpha - 1}, x > 0$, determine the M.L.E. of θ based on a random sample of size n. (α known).

4. Let $X_1, X_2, ..., X_n$ be a random sample from beta distribution of first kind with parameters μ and 1. Find the M.L.E. of $\theta = \frac{\mu}{\mu + 1}$.

5. Construct $100(1 - \alpha)\%$ confidence interval for the variance of normal distribution with unknown mean.

6. Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, σ^2 known. Derive the BCR of size α for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1 (> \mu_0)$.

7. Derive maximum likelihood ratio test procedure for testing equality of variances of normal populations, with unknown mean.

8. Stating the assumptions clearly describe students t test for paired samples.

9. Write a note on Chi-square test for testing independence of attributes.

PART – C

Answer **any three** of the following : **(3×10=30)**

10. a) Explain the moment method of estimating the parameters. **4**

b) Show that the sum of the items of a random sample of size n from an exponential distribution with p.d.f. $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ is a sufficient statistic, for θ. **6**

11. a) Explain Neyman Pearson Lemma. **4**

b) Derive the most powerful test procedure for testing

$H_0 : f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$ against $H_1 : f(x) = \frac{1}{\pi} \frac{1}{1+x^2}, -\infty < x < \infty$.

And find its power. **6**

12. Derive the likelihood ratio test procedure for testing $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$, where σ_1^2 and σ_2^2 are the variances of two normal populations with unknown means.

13. a) Two samples of sizes N_1, N_2 have respectively frequencies $f_1, f_2, ..., f_n$ under the same headings. Show that Chi-square for such a distribution is equal to $\sum_{r=1}^{n} N_1 N_2 \left[\frac{\left(\frac{f_r}{N_1} - \frac{f_r'}{N_2} \right)^2}{f_r + f_r'} \right]$. **6**

b) Write a note on Yates correction for continuity. **4**
