Reg. No. | | | | | | | | | |

# MTE 451

**II Semester M.Sc. Degree Examination, September/October 2022**
**(CBCS -New Syllabus)**
**MATHEMATICS**
**Discrete Mathematics and Applications (Open Elective)**

Time : 3 Hours Max. Marks : 70

***Instructions :*** *1) Answer **any five full** questions.*
*2) No **additional** sheets will be provided for answering.*
*3) Use of scientific calculator is **permitted**.*

1. a) Let a, b be integers with $a \equiv 4 \pmod{13}$ and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ and satisfying (i) $c \equiv a + b \pmod{13}$ and (ii) $c \equiv a^2 + b^2 \pmod{13}$.
   b) What time does a 12-hour clock read
      i) 80 hours after it reads 11 : 00 ?
      ii) 12 hours after it reads 06 : 00 ?
   c) Find the octal representation of $(2022)_{10}$ and $(D5A7)_{16}$. **(6+4+4)**

2. a) Let a = bq + r, where a, b, q and r are integers. Then prove that gcd (a, b) = gcd (b, r).
   b) Find the greatest common divisor of 124 and 323 using the Euclidean algorithm.
   c) Encrypt the message STOP using the RSA cryptosystem with key (2537, 13). Note that 2537 = 4359, p = 43 and q = 59 are primes. **(5+4+5)**

3. a) A student can choose a computer project from one of three lists. The three lists contain 23, 15 and 19 possible projects, respectively. No project is on more than one list. Find the number of possible projects to choose from.
   b) A playoff between two teams consists of at most five games. The first team that wins three games wins the playoff. Find the number of different ways for the playoff.
   c) Explain r-permutation and r-combination of elements of a set. How many permutations of the letters ABCDEFGH contain the string ABC ? **(3+6+5)**

**P.T.O.**

4. a) State the Binomial theorem. Find the coefficient of $x^5y^7$ in the expansion of $(2x + y)^{12}$.

   b) Determine the number of non-negative solutions of the equation $a + b + c \leq 10$.

   c) Suppose that a computer science laboratory has 15 workstations and 10 servers. A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time. We want to guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. Although we could do this by connecting every workstation directly to every server (using 150 connections), find the minimum number of direct connections needed to achieve this goal. **(4+5+5)**

5. a) Determine the number of poker hands of five cards from a standard deck of 52 cards. Also find the number of ways to select 47 cards from a standard deck of 52 cards.

   b) Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. Find the number of bit strings of length five.

   c) Using generating functions find the number of different ways of distributing eight identical cookies among three distinct children if each child receives at least two cookies and not more than four cookies. **(4+5+5)**

6. a) Define the following and illustrate each with example.

   i) Symmetric relation

   ii) Asymmetric relation

   iii) Antisymmetric relation

   b) Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (1, 1), (1, 3), (2, 4), (3, 2)\}$. Draw the digraph of R. Find $R^2$, and $R^\infty$. Show that $M_R \odot M_R = M_{R^2}$.

   c) Show that a partition of a set induces an equivalence relation and every equivalence relation gives a partition of a set. **(5+4+5)**

7. a) Let $D_n$ denote the set of all positive divisors of n. Show that the relation aRb if and only if a|b for all a, b $\in D_n$, is a partial order on $D_n$. Draw the Hasse diagram of the poset $D_{24}$.

   b) Show that in a Boolean algebra, for any a, b and c.

      i) If $a \leq b$, then $a \vee c \leq b \vee c$.

      ii) If $a \leq b$, then $a \wedge c \leq b \wedge c$.

   c) Consider the Boolean function $f(x_1, x_2, x_3) = (x_1 \vee x_2) \wedge (x_1 \wedge (x'_2 \vee x_3))$. Construct the truth table for $f : B_3 \rightarrow B$ determined by this Boolean function. **(5+4+5)**

8. a) Define the group $G \times H$ of direct products of two groups G and H. Find the order of the group $z_2 \times z_2 \times z_2$.

   b) Let $e : B^m \rightarrow B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a nonzero code word.

   c) Twelve digit bar codes use the twelfth digit as a check digit by choosing it so that the sum of the digits in even numbered positions and three times the sum of the digits in odd numbered positions are congruent to 0 mod 10. Show that this code will detect a single error in an even numbered position but may not detect two errors in even numbered positions. **(4+5+5)**