Reg. No. | | | | | | | | | |

MTS 554

IV Semester M.Sc. Degree Examination, September/October 2022
MATHEMATICS
Partial Differential Equations
(CBCS – New Syllabus)

Time : 3 Hours Max. Marks : 70

Note : *1) Answer **any five full** questions.*
*2) Answer to **each** full question shall **not** exceed **eight** pages of the answer book. **No** additional sheets will be provided for answering.*
*3) **Use** of scientific calculator is **permitted**.*

1. a) Find the orthogonal trajectories on the conicoid $(x + y)z = 1$ of the conics in which cut by the system of planes $x - y + z = c$, c is a parameter.

   b) Find the general solution of the partial differential equation $x^2p + y^2q = u(x+y)$.

   c) Construct the partial differential equation by eliminating the arbitrary function F from the equation $F(x^2 + y^2 + u^2, u^2 - 2xy) = 0$. **(7+5+2)**

2. a) Prove that the necessary and sufficient condition that the Pfaffian differential equation $p(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)\, dz = 0$ to be integrable is that curl $X = 0$, where $X = (P, Q, R)$.

   b) Find the general solution of the equation $x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u$. Hence find the integral surface through the curve $x + y = 0$, $u = 1$. **(7+7)**

3. a) Verify $z = ax + by + a + b - ab$ is a complete integral of the partial differential equation $z= px + qy + p + q - pq$, where a and b are arbitrary constants, show that the envelope of all planes corresponding to the complete integrals provides a singular solution of the partial differential equation. Determine the general solution by finding envelope of those planes that pass through the origin.

   b) Find the surface which is orthogonal to the system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $(x^2 - y^2) = a^2$. **(7+7)**

4. a) Solve the semi-linear equation $xu_x + yu_y = u + 1$ with Cauchy data $u(x, 0) = x^2$, using Cauchy method of characteristics.

   b) Show that the equation $xp = yq$ and $x^2 + q - xz = 0$ are compatible and solve. **(7+7)**

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5. a) Solve the non-linear equation $(xp^2 + yq) = z$ which passes through the curve $y = 1$, $x + z = 0$, using Charpits method.

   b) Solve : $(D^2 + DD' - 2(D')^2)u = e^{(x+y)}$.

   c) Solve : $(D^2 - D')u = A\cos(lx + my)$. **(6+4+4)**

6. a) Find the nature of the equation $u_{xx} + 2u_{xy} + u_{yy} = \sin x + e^x \log(y\cos y)$ and reduce it into its canonical form.

   b) Give the classification for a second order semi linear equation in two independent variables x and y for a single unknown function u(x, y). Prove that the classification is invariant under a one-one transformation having non-zero Jacobian determinant. **(7+7)**

7. a) Obtain the solution of diffusion equation in spherical co-ordinate system.

   b) Find the solution of $u_{tt} = c^2 u_{xx}$ in the interval $[0, l]$ subjected to boundary conditions $u(0, t) = u(l, t) = 0$ and the initial condition,

$$u(x,0) = \begin{cases} \varepsilon \dfrac{x}{b} & 0 \le x \le b \\ \varepsilon \dfrac{x-l}{b-l} & b \le x \le l \end{cases}$$ and $u_t(x, 0) = 0$, $t \ge 0$. **(7+7)**

8. a) Derive D'Alembert's solution of the one dimensional wave equation $u_{tt} = c^2 u_{xx}$, $-\infty < x < \infty$, $t \ge 0$ subject to the initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ $\forall -\infty < x < \infty$, where f(x) and g(x) are c^2 functions.

   b) Derive Riemann-Volterra solution of the equation $u_{tt} - c^2 u_{xx} = F(x, t)$, $-\infty < x < \infty$, $t \ge 0$ subject to the initial conditions as in 8a. **(6+8)**
