Reg. No. | | | | | | | | | |

MTS 556

IV Semester M.Sc. Degree Examination, Sept./Oct. 2022
(CBCS – New Syllabus)
MATHEMATICS
Advanced Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Note : *1) Answer **any five full** questions.*
*2) No additional sheets will be **provided** for answering.*
*3) **Use** of scientific calculator is **permitted**.*

1. a) Let $n \leq m$ be positive integers. Give a combinatorial proof of the identity :

$$\binom{m}{0}\binom{n}{0}+\binom{m}{1}\binom{n}{1}+\binom{m}{2}\binom{n}{2}+\ldots+\binom{m}{n}\binom{n}{n}=\binom{m+n}{n}.$$

b) In the parliament of a certain country there are 248 seats and three political parties. How many ways can these seats be divided among the parties such that no single party has a majority ?

c) How many anagrams of the letters of 'ABRACADABRACADABRA' are there so that no two 'B' are adjacent ?

d) How many triangles can be drawn, all of whose vertices are the vertices of given octagon and all the sides of the triangle are diagonals of the octagon ? **(4+4+3+3)**

2. a) State the Pigeonhole principle. Suppose that a computer science laboratory has 15 workstations and 10 servers. A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time. We want to guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. Although we could do this by connecting every workstation directly to every server (using 150 connections), what is the minimum number of direct connections needed to achieve this goal ?

b) If there are 10 people in a party, then show that there will be either 4 mutual friends or there will be three mutual strangers.

c) What is a derangement ? If D_n is number of derangements of {1, 2, ..., n}, then show that $D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ for all $n \in \mathbb{N}$. **(5+4+5)**

3. a) What are Fibonacci numbers ? Show that the n^{th} Fibonacci number is given by

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

b) Solve the recurrence relation $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$; with initial conditions $a_0 = 1$, $a_1 = 4$ and $a_2 = 8$.

c) Find the number of integer solutions of $x_1 + x_2 + x_3 = 12$ satisfying the conditions $-2 \le x_1 \le 6$, $x_2 \ge 3$ and $x_3 \ge 0$. **(5+5+4)**

4. a) Define the Stirling number of second kind $\begin{bmatrix} n \\ k \end{bmatrix}$.

Prove that :

i) $\begin{bmatrix} n \\ 2 \end{bmatrix} = 2^{n-1} - 1$

ii) $\begin{bmatrix} n \\ r \end{bmatrix} = r\begin{bmatrix} n-1 \\ r \end{bmatrix} + \begin{bmatrix} n-1 \\ r-1 \end{bmatrix}$

b) Prove that, the number of permutations of S_n in the conjugacy class of S_n corresponds to the cycle structure $1^{l_1} 2^{l_2} \ldots n^{l_n}$ is $\frac{n!}{l_1! \cdot l_2! \ldots l_n! \cdot 1^{l_1} \cdot 2^{l_2} \ldots n^{l_n}}$. Find the number of distinct conjugacy class of S_4 and using the theorem find the number of elements in each conjugacy class of S_4.

c) Define the cycle index polynomial of a subgroup of a permutation group. Obtain cycle index polynomial of dihedral group D_6. **(4+6+4)**

5. a) State Poly's enumeration theorem. Suppose we are given 8 similar spheres in three different colours, say, three red, two blue and one yellow (spheres of the same color being indistinguishable). In how many ways can we distribute the six spheres on the 8 vertices of a cube freely movable in space ?

   b) If the square regions of the 3×3 square grid are coloured using 3 colours Red, Blue and Yellow, then find the number of possible colour patterns. How many of these colour patterns contains at least 3 Blue coloured regions ? **(7+7)**

6. a) Define a Boolean Algebra. Prove that a Boolean algebra is associative under the operations of + and $\cdot$.

   b) Reduce the expression '$f = (x \oplus yz) + \overline{(\bar{x}\,\bar{y} \oplus w)} + \bar{x}yz$' to minimal sum-of-products expression.

   c) Design a combinational circuit that accepts a 3-bit number and generates an output binary number equal to the square of the input. **(4+4+6)**

7. a) Minimize the switching function : $\sum m(2, 3, 4, 7, 10, 12, 13, 15)$.

   b) Minimize $f = (\bar{A} + B + \bar{C})(\bar{A} + B + C)$ using the map technique.

   c) State and prove maximum flow minimum cut theorem. **(4+3+7)**

8. a) Among simple k-chromatic graphs (that is, k-partite) with n vertices, prove that the Turan's graph $T_{n,k}$ is the unique graph with the maximum number of edges.

   b) Write the Dijkstra's shortest path algorithm and illustrate with suitable example. **(6+8)**
