Reg. No. | | | | | | | | | |

STH 452

II Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Distribution Theory

Time : 3 Hours Max. Marks : 70

***Note** : Question No. **1** is **compulsory**. Answer **any four** questions from the remaining seven questions.*

1. Answer **any six** sub-divisions. **Each** question carries **3** marks. **(6×3=18)**

   a) Find the m.g.f. of normal random variable with parameters μ and σ^2.

   b) If X and Y have joint pdf $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Examine whether X and Y are independent.

   c) Define non-central t distribution. State its mean and variance.

   d) Prove that, $V(X) = V(E(X|Y)) + E(V(X|Y))$.

   e) Let $X \sim U(0, 1)$. Find the transformation $Y = g(X)$ which has the density function $f_y(y) = e^{-y}$, $y > 0$.

   f) Prove or disprove : Exponential distribution satisfies lack-of-memory property.

   g) Define Wishart distribution and mention any one of its properties.

   h) Let $Y_1, Y_2, \ldots, Y_n$ are order statistics of a random sample from an exponential distribution with parameter λ. Find the joint distribution of Y_1, Y_n.

Answer **any four full** questions from the following, **each** question carries **13** marks. **(4×13=52)**

2. a) Obtain p.g.f. of a negative binomial distribution, show that negative binomial is a members of power series distribution.

   b) State and establish any two properties of Weibull distribution. **(7+6)**

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3. a) If X is any continuous random variable with cdf F. Find the distribution of $Y = F(X)$.

   b) Define truncated distribution. Write down the probability mass function of a Poisson distribution truncated at zero and find its mean and variance.

   c) What is probability integral transformation ? What is its use ? **(5+5+3)**

4. a) Let X and Y be iid gamma random variables. Find the joint distribution of $U = \frac{X}{X+Y}$ and $V = (X + Y)$. Obtain the conditional distribution of V given U = u.

   b) Show that a random variable X has standard Cauchy distribution if and only if $\frac{1}{X}$ also has standard Cauchy distribution.

   c) Define extreme value distribution and state its important properties. **(6+4+3)**

5. a) Prove or disprove : Mean and variance of a random sample of a normal distribution are independent.

   b) Define logistic distribution. Obtain its moment generating function. **(8+5)**

6. a) Let $Y = (Y_1, Y_2, \ldots, Y_n)'$ where Y_i's are iid N(0, 1) random variables. Prove that a necessary and sufficient condition that $Y'AY$ has chi-square distribution is that A is idempotent.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential with parameter θ. Let $Y_1, Y_2, \ldots, Y_n$ be order statistics. Then show that $Y_s - Y_r$ and Y_r are independent. **(5+8)**

7. a) Derive the characteristic function of Wishart distribution.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that $\sum_{i=1}^{n}(X_i - \bar{x})^2 / \sigma^2$ has χ^2-distribution with n-2 degrees of freedom. **(7+6)**

8. a) Define multivariate normal distribution. Find the distribution of linear combinations of components of a vector having multivariate normal distribution.

   b) Prove or disprove : The conditional distribution of any sub-vector given the other components in a multivariate normal random vector is again multivariate normal. **(7+6)**
