Reg. No. | | | | | | | | | |

STH 453

II Semester M.Sc. Examination, September/October 2022
STATISTICS
Theory of Point Estimation

Time : 3 Hours   Max. Marks : 70

Note : *Question No.* ***1*** *is* ***compulsory****. Answer* ***any four*** *questions from the remaining seven questions. Figures to* ***right*** *indicate marks to sub-questions.*

Answer **any 6** of the following : **(6×3=18)**

1. a) Demonstrate that B(1, 2/2) is not complete.

   b) State the likelihood principle of obtaining minimal sufficient statistics.

   c) Given a sample of size n from $N(\theta, 1)$, obtain Fisher information about θ.

   d) Based on a random sample of size n from $Exp(\lambda)$, obtain Moment estimator of $\lambda > 0$.

   e) Prove that UMVU estimator is unique.

   f) Define consistent estimator and state invariance property of consistent estimator.

   g) Let $X_1, X_2 ..., X_n$ be a random sample of size n from Poisson distribution with parameter θ. Show that sample mean is CAN estimator for θ.

   h) Given a random sample from $U(0, \theta)$ obtain MLE of θ.

2. a) Define Fisher information for one and several parameter models. Obtain the Fisher information contained in a sample of size n from Cauchy distribution with median θ.

   b) Explain the concept of completeness. Is the Bernoulli family complete ? Justify your answer. **(7+6)**

3. a) Obtain a sufficient statistics for θ of $U(0, \theta)$ distribution and examine whether it is complete.

   b) State the factorization theorem and prove it in the discrete case. **(6+7)**

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4. a) State and Prove Rao-Blackwell-Lehman-Scheffe theorem.

   b) Given a random sample of size n from Poisson distribution with parameter λ. Let N be the number of observations which are equal to zero. Obtain unbiased estimator for $e^{-\lambda}$ and improve the estimator using Rao-Blackwell Theorem. **(5+8)**

5. a) State and Prove Cramer-Rao inequality. Use it obtain the UMVUE for θ based on a random sample of size n from exponential distribution with mean θ.

   b) State and Prove necessary and sufficient conditions for an estimator to be UMVUE. **(8+5)**

6. a) State and prove invariance property of ML estimator.

   b) Under the regularity conditions prove that MLE is a consistent estimator for θ. **(7+6)**

7. a) Given a random sample of size n from a lognormal distribution with parameters μ and σ, obtain moment estimators for μ and σ.

   b) Given a set of iid r.v's $X_1, X_2, ..., X_n$ from an exponential distribution with location parameter θ and scale parameter σ. Obtain MLE's of (μ, σ). **(6+7)**

8. a) Given a random sample of size n from a Gamma $(\alpha, 1)$ distribution, obtain MLE's of $(\alpha, 1)$. Show that the estimators is CAN.

   b) Given a set of iid r.v's $X_1, X_2, ..., X_n$ from a Binomial (n, p) distribution, where both n and p are unknown, obtain moment estimators of (n, p). Prove or disprove that moment estimators are jointly consistent for (n, p). **(5+8)**
