Reg. No. [ ]

STS 455

II Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Actuarial Statistics

Time : 3 Hours Max. Marks : 70

Note : *1) Question No.* ***1*** *is* ***compulsory****.*
2) Answer ***any four*** *questions from the remaining* ***seven*** *questions.*

1. Answer **any six** questions. **(6×3=18)**
   a) Give an economic justification for insurance system with example. Also discuss about its limitations.
   b) Define distribution function and survival function of T(x). Derive these in terms of survival function.
   c) Derive an expression for curtate expectation life.
   d) What is select life table ? When it is used ?
   e) Suppose that Gompertz' law applies with $\mu_{30} = 0.000130$ and $\mu_{50} = 0.000344$. Calculate ${}_{10}p_{40}$.
   f) Differentiate between n-year pure endowment insurance and n-year endowment insurance.
   g) Explain annuity certain and annuity due and give their expressions.
   h) Describe fully continuous premiums, fully discrete premiums.

2. a) Derive $S_x(t)$ and force of mortality under Makeham's law of mortality.
   b) Suppose the survival function of life length random variable is $S(x) = 1 - x^2/100$ for $0 < x \leq 10$. Find
      i) density function of life length random variable;
      ii) density function of T(x)
      iii) ${}_3p_3$
      iv) complete expectation of life.
   c) Let the probability distribution function of length of life be $F_0(x) = 1 - (1 - x/120)^{1/6}$ for $0 < x \leq 120$, calculate e^0_x. **(4+6+3)**

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3. a) Show that the density of T(x) can be written $f_{Tx}\ t = {}_tp_x\mu_{x+t}$ and also for $\mu_{x+t} = t$ for $t \geq 0$. Calculate ${}_tp_x\ \mu_{x+t}$ and e^0_x.

   b) Define time until death for a person age x. Let $F_0\ t = 1 - (1 - t/120)^{1/6}$, for $0 \leq t \leq 120$. Calculate the probability that

      i) a life aged 30 dies before age 50 and

      ii) a life aged 40 survives beyond age 65.

   c) A life aged (40) is subject to an extra risk for the next year only. Suppose the normal probability of death is 0.004, and that the extra risk may be expressed by adding the function 0.03 (1 – t) to the normal force of mortality for this year. What is the probability of survival to age 41 ? **(5+5+3)**

4. a) What are the fractional age assumptions ? Show that under the assumption of uniform distribution of deaths in the year of death that K(x) and T(x) – K(x) are independent and that T(x) – K(x) has the uniform distribution on the interval (0, 1). Prove the equivalence of UDD1 and UDD2.

   b) The Gompertz law of mortality is defined by the requirement that $\mu_t = Ac^t$ for some constants A and c. What restrictions are there on A and c for this to be a force of mortality ? Write an expression for ${}_tp_x$ under Gompertz law.

   c) Given that q70 = 0.01422 and q71 = 0.01310, calculate, 0.7 q 70.6 assuming a uniform distribution of deaths. **(6+5+2)**

5. a) Part of a select life table with two year selection period is given below.

[x]	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	x + 2
30	9907	9905	9901	32
31	9903	9901	9897	33
32	9899	9896	9893	34

   Calculate (i) 2p[32], (ii) 2q[30]+1, (iii) 2|q [31], (iv) 2q32, (v) 2|2q[30].

   b) If l_0 is 1,00,000 and $S(x) = 1/(1 + x^2)$, calculate l_x by random survivorship method for x = 15.

   c) For an amount A invested today one gets Rs. 3,00,000 after six years with interest compounded quarterly with nominal interest rate $i^{(4)} = 0.08$. What is A ? **(5+5+3)**

6. a) Define n-year endowment insurance. Show that $\bar{A}_{x:\overline{n|}} = \bar{A}^{1}_{x:\overline{n|}} + \bar{A}_{x:\overset{\ \ 1}{\overline{n|}}}$.

   b) Define deferred insurance. Prove that $_{u|}\bar{A}^{1}_{x:\overline{n|}} = \bar{A}^{1}_{x:\overline{u+n|}} - \bar{A}^{1}_{x:\overline{u|}}$.

   c) Consider an insurance policy issued to (x) under which the death benefit is $(1 + j)^t$ if death occurs at age x + t, with the death benefit being payable immediately on death.

      i) Derive an expression for the actuarial present value of the death benefit if the policy is an n-year term insurance.

      ii) Derive an expression for the actuarial present value of the death benefit if the policy is a whole life insurance. **(4+5+4)**

7. a) Describe in words the benefits with the present values given and write down an expression in terms of actuarial functions for the expected present value.

$$Y_1 = \begin{cases} \bar{a}_{\overline{T_x|}} & \text{if } T_x \le 15, \\ \bar{a}_{\overline{15|}} & \text{if } T_x > 15. \end{cases}$$

and $$Y_2 = \begin{cases} a_{\overline{15|}} & \text{if } 0 < K_x \le 15, \\ a_{\overline{K_x|}} & \text{if } K_x > 15 \end{cases}.$$

   b) An annuity immediate pays an initial benefit of one per year, increasing by 10.25% every four years. The annuity is payable for 40 years. If the effective interest rate is 5% find an expression for the present value of this annuity.

   c) Explaining the terms involved, show that $\sum_{k=0}^{\infty} \ddot{a}_{k+1}\, {}_{1|k}q_x = \sum_{k=0}^{\infty} v^k\, {}_k p_x$.

(5+5+3)

8. a) An insurer issues a 25-year annual premium endowment insurance with sum insured \$100000 to a select life aged 30. The insurer incurs initial expenses of \$ 2000 plus 50% of the first premium and renewal expenses of 2.5% of each subsequent premium. The death benefit is payable immediately on death. (i) Write down the gross future loss random variable. (ii) Calculate the gross premium using the standard select survival model with 5% per year interest.

   b) If the life length random variable X has a uniform distributed over (0, 120), determine the actuarial value of the benefit paid under continuous case. Also, determine the variance of present value of the benefit. **(8+5)**