Reg. No. | | | | | | | | | |

BSCMTEN 201

II Semester Open Elective (NEP 2020) Examination, September 2022
(2021 – 2022 Batch Onwards)
MATHEMATICS – II (Science Stream)

Time : 2 Hours Max. Marks : 60

Instructions : *1) Answer **any eight** questions from Part – **A**. **Each** question carries **3** marks.*

*2) Answers to Part – **A** should be written in the **first few** pages of the answer book before answers to Part – **B**.*

*3) Answer **any six** questions from Part – **B** by choosing **two** questions from **each** Unit. **Each** question carries **6** marks.*

*4) Scientific calculators are **allowed**.*

PART – A

Answer **any eight** questions. **(8×3=24)**

1. Define a binary operation on the set of positive integers by a * b = max {a, b}. Show that the operation is both associative and commutative.

2. On the set of integers Z, * is defined by $a + b - 1 \ \forall\ a, b \in Z$. Find the inverse of 2.

3. Form the table for multiplication modulo 10 for the set {1, 3, 7, 9}.

4. Construct the Klein's four group.

5. Evaluate : $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy - y - 2x + 2}{x - 1}, x \neq 1$.

6. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for $f(x, y) = x^2 + 3xy + y - 1$.

7. Find $\dfrac{dw}{dt}$ for $w = x^2y - y^2$ with $x = \sin t$ and $y = e^t$.

8. Prove that $f(x, y) = \dfrac{\sqrt{x} + \sqrt{y}}{x + y}$ is a homogeneous function of degree $-\dfrac{1}{2}$.

P.T.O.

9. Evaluate $\int_C (x+y)\,ds$, where C is the straight line segment x = t, y = 1 – t, z = 0 from (0, 1, 0) to (1, 0, 0).

10. Evaluate $\int_0^3 \int_0^2 \left(4-y^2\right) dy\, dx$.

11. Sketch the region of integration $\int_0^{\pi} \int_0^{x} (x \ \sin y)\, dy\, dx$.

12. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dxdydz$.

PART – B

Answer **any six** questions by choosing **two** questions from **each** Unit. **(6×6=36)**

Unit – I

13. On the set of all integers Z, define a binary operation * by a*b = a + b + 1, a, b $\in$ R. Prove that (Z, *) is an abelian group.

14. Let G = Q – {–1} be the set of all rational numbers except – 1 and * be a binary operation on G defined by a * b = a + b + ab $\forall$a, b $\in$ G. Prove that (G, *) is an infinite abelian group.

15. Prove that the set G = {3, 6, 9, 12} forms a finite abelian group of order 4 under multiplication modulo 15.

16. Find the orders of the elements of the additive modulo 6 group $(Z_6, +_6)$.

Unit – II

17. Show that the function $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches (0, 0) using two path test.

18. Show that f (x, y, z) = $x^2 + y^2 - 2z^2$ satisfies Laplace equation.

19. Find the local extreme values of the function f (x, y) = $xy - x^2 - y^2 - 2x - 2y + 4$ if exists.

20. Verify Euler's theorem for the function f (x, y) = $x^2 + 2hxy + y^2$.

Unit – III

21. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).

22. Sketch the region of integration of the double integral $\int_0^1 \int_2^{4-2x} dy\,dx$ and write an equivalent double integral with the order of integration reversed and evaluate.

23. Find the area bounded by $y = x$ and $y = x^2$ in the first quadrant.

24. Evaluate :

    a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\,dy\,dx$

    b) $\int_0^1 \int_0^1 \int_0^1 \left(x^2 + y^2 + z^2\right) dzdydx$.